OPTIMAL DESIGN OF CAR SUSPENSION SYSTEM USING GENETIC ALGORITHM

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To

My Father

CERTIFICATE

It is certified that the work contained in the thesis entitled 'OPTIMAL DESIGN OF CAR SUSPENSION SYSTEM USING GENETIC ALGORITHM' by Vikas Saxena is carried out under my guidance and it has not been submitted elsewhere for the award of the degree.

Kalyanmoy Deb

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Abstract

The comfort in car riding largely depends on its suspension system. Isolation of passenger compartment from the vibrations due to road undulations is the primary purpose of the suspension system. Two-dimensional as well as three-dimensional dynamic models of the suspension system have been developed with front and rear suspensions as of independent type. Springs and dampers of the suspension system can be chosen to make the isolation maximum, while satisfying other important criteria of the performance of the system. The optimization for the design of suspension system is carried out using genetic algorithm— a optimization technique based on the principles of natural genetics. Various idealized road conditions are used with different objectives related to different motions of the sprung mass of the car. to get the optimal solution. Vertical jerk experienced by the passenger is kept below a certain level by using a constraint on its value. Natural frequencies of the subsystem of the suspension system imposes other constraints on the optimal solution. A realistic road is modeled using a polyharmonic function. ISO standards are used to find the level of comfort based on the acceleration levels on different frequencies. The solution found in all the cases are better than the present design of TELCO, Pune. Success of genetic algorithm in finding the near optimal solution in highly complex, multimodal and disjointed feasible search space of the optimal suspension design problem suggests their immediate application in other problems of engineering.

CONTENTS

Chapter 1

Introduction

In today's competitive world the need of the time is to use the available resources in a most effective manner. This means that, whenever possible, the maximization of the performance of any system should be achieved. In engineering design problems, the measure of the performance varies from problem to problem. The maximization of performance may be in terms of reducing overall cost of the production or it may be in terms of maximization of the profit for a product [7]. This should be achieved with other related aspects of the problem in mind. In simpler words, the performance should be optimal taking care of all conflicting aspects of the problem.

The modern-day vehicle is a complex assembly created to meet a variety of design objectives, including fuel economy, handling, performance, safety, passenger comfort, driveability, and ride quality. No single vehicle can meet all of these objectives maximally. As a result, designers make compromises on some objectives while emphasizing some other design objectives. Suspension system of any vehicle has to meet various requirements for the proper and smooth functioning of the vehicle. Suspension is supposed to support the vehicle weight and to provide proper handling characteristics of the vehicle. Most importantly, it attempts to isolate the passenger compartment of the vehicle form vibrations due to road roughness. The effect of road vibrations experienced by the passenger interferes with the comfort during the ride. The perception of the ride comfort is a subjective matter but in general vehicle designed for good comfort should have a suspension system which reduces the transmissibility of the road undulations to its occupants.

In this thesis, we attempt to find an optimal design of suspension system of a car with the comfort of the passenger as the objective. The dynamic model of the suspension system is achieved mathematically to simulate the problem. Firstly, a two-dimensional model is developed and used for solving the problem. Thereafter, a three-dimensional model is developed for more detailed study of a car's motion and for getting a better design. Different motions of the car are taken into consideration and objective functions are formulated. The jerk experienced by the occupants is taken as a constraint. Natural frequencies of the front and rear subsystems of the suspension system also impose some constraints to the optimal problem.

To solve the optimization problem, the genetic algorithm is used. Genetic algorithms are new yet potential techniques based on the principles of natural genetics [6,12,14]. Population-based

approach of the genetic algorithms enables them to find the global optimum solution in most complex problems. Operators of these algorithms have remarkable ability of combining information of two partial solutions to get a better solution [14]. In this thesis, GA technique is tested well for finding optimal solution for different objectives and for different road excitations.

ISO provides standards on human exposure to whole-body vibrations [15,20]. Some limits have been discussed in these standards for the comfort in such conditions. Motion of a car over a rough road gives this kind of exposure to its occupants. An attempt has been directed towards getting an optimal design in accordance of these standards.

In Chapter 2, we develop the theory of the suspension design procedure, with two as well as three-dimensional conditions. Chapter 3 gives an overview of the genetic algorithms discussing its working principle and properties of its basic operators. Some previous applications of GAs to engineering problems are also presented. The optimal design problem formulations for different objectives related to the suspension design are presented in Chapter 4. This chapter also presents the formulation of the constraints related to natural frequencies of subsystems of the suspension system. Chapter 5 presents the results of different simulation runs of a traditional optimization method and of a GA for finding optimal solutions. Simulation of the car motion over a realistic road and subsequent analysis in accordance with ISO 2631 are also discussed. Conclusions and extensions of the present work are presented in Chapter 6.

Chapter 2

Suspension Systems: Theory And Modeling

The suspension systems, like other parts of the vehicle, are designed to meet specific requirements. These requirements consider the number of passenger to be carried, the useful load, the type of the ride desired and the required handling characteristics. The primary function of the suspension system is to isolate the structure, as far as possible, from shock loading and vibrations due to irregularities of the road surface. Secondly, it must do this without impairing the stability, steering or general handling qualities of the vehicle. The primary requirement is met by the use of flexible elements and dampers while the second is achieved by use of mechanical linkages [17]. As is generally true, when one suspension property is improved, the quality of another property is reduced. In this thesis, we concentrate on designing the former for given road conditions.

2.1 Vehicle Dynamics

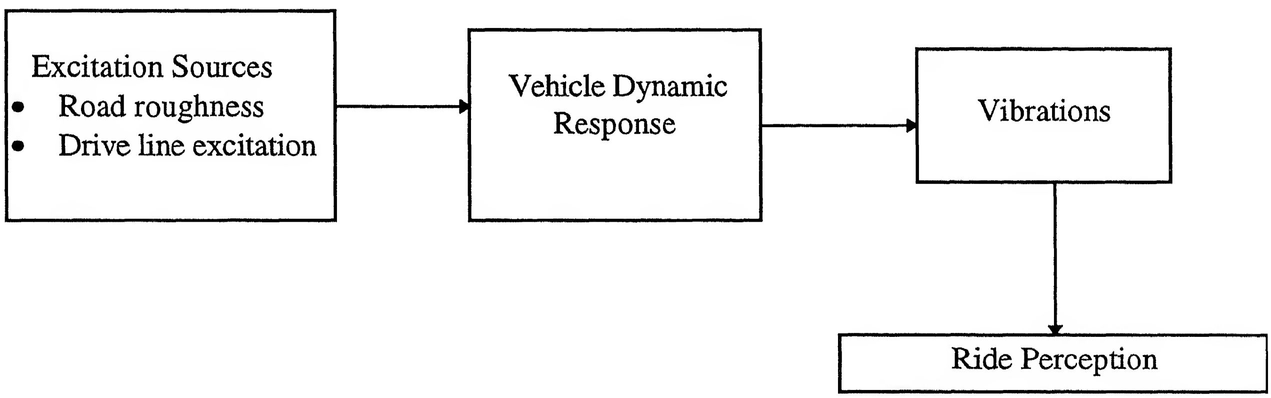
Progress on understanding the ride motion of a vehicle is made by applying the principles of vibration theory. However, putting this theory into practice is not so straightforward, since it includes several suspension design issues. The designer has to achieve good ride comfort for the driver and passengers, acceptable control of the body attitude and adequate control of the dynamic tyre loads. The diameter of the tyre, size of the contact patch between tyre and the road, the rate of tyre acting as a spring, and weight of the wheel and axle assembly affect the magnitude of the shock transmitted to the axle, while amplitude of wheel motion is influenced by all these factors plus the rate of suspension springs, damping effect of the shock absorbers and weight of sprung mass and unsprung mass. The *unsprung mass* can be loosely defined as that between the road and the main suspension springs, while the *sprung mass* is that supported on these suspension springs [10,17].

Dampers in suspension system have a two-fold function. First, they are for reducing the tendency for carriage unit to continue to bounce up and down on its springs after the disturbance that causes the initial motion has ceased. Secondly, they prevent excessive building-up of the amplitude of the bounce as a result of periodic excitation at a frequency identical to the natural frequency of the vibration of the system. Thus, for maximum ride comfort, proper spring stiffness and damper characteristics must be chosen.

2.1.1 Ride Characteristics

The vehicle is a dynamic system but only exhibits action response to an excitation input. The response properties determine the magnitude and direction of vibrations imposed to the passenger compartment and ultimately determines the passenger's perception of the vehicle. The understanding of ride involves the study of following [11] :

- Ride excitation sources
- Basic mechanics of the vehicle vibration response
- Human perception and tolerance of vibration



2.1.1.1 Excitation Sources

There are multiple sources from which vehicle ride vibrations may be caused. These are generally divided into factors such as road roughness and on-board sources. The road roughness comes from the roughness of the road, while the on board sources arise from reciprocating

components of the car. However, in this thesis we shall concentrate on the factors related to road roughness only.

Road roughness encompasses everything from potholes to the ever present random deviations reflecting the practical limits of precision to which the road surfaces can be constructed and maintained. Roughness is described by the elevation profile along the wheel tracks over which the vehicle passes. Road profiles fit the general category of the "broad-band random signals" and can be described by the profile itself or its statistical properties [11].

Like any random signal, the elevation profile measured over a length of road can be decomposed using the Fourier Transform into a series of sine waves varying in their amplitudes and phase relationship. In chapter 5, we simulate the road by assuming it as an addition of sine waves defined in varying wavelengths.

2.1.1.2 Vehicle Response Properties

A systematic treatment of the vehicle as a dynamic system begins by identifying basic supporting structure of a vehicle on its suspension system, i.e., the motion of the body and axles. At low frequencies the body, which is considered to be the sprung mass portion of the vehicle, moves as an integral unit of the suspension. This is like rigid body motion. The axles and associated hardware, which form the unsprung masses also moves as a rigid body and consequently impose excitation forces on the sprung mass.

The dynamic behavior of the vehicle can be characterized most meaningfully by considering the input-output relationships. This input may be any one of the excitations discussed

earlier. The output most commonly of interest will be the vibrations on the body. The ratio of output and input amplitudes represents "gain" for the dynamic system. The term *transmissibility* is often used to denote this gain. Transmissibility is a non-dimensional ratio of the maximum amplitude of vibration of the sprung mass to the amplitude of excitation for a system in forced vibration. Although this ratio may be calculated for amplitudes of forces, displacements, velocities or accelerations, we have considered amplitude of linear and angular displacements.

2.1.1.3 Perception of Ride

Theoretical assessment of the ride vibrations must deal with the issue of how a ride is perceived. For that purpose, one must first attempt to define ride. Ride is a subjective perception, normally associated with the level of the comfort experienced when traveling in a vehicle. Therefore, in its broadest sense, the perceived ride is the cumulative product of many factors. The tactile vibrations transmitted to the passenger's body through seat, and at the hand and the feet, are factors most commonly associated with the ride. Yet it is often difficult to separate the influences of acoustic vibrations (noise) in the perception of a ride. Additionally, the general comfort level can be influenced by seat design and its fit to the passenger, temperature, ventilation, interior space, hand holds, and many other factors. These factors may contribute to what might be termed as the "ride quality" of a vehicle [11]. Some of the above factors, such as vibrations, can be measured objectively, while others, such as seat comfort, are still heavily dependent on the subjective evaluation methods. Measurement of vibrations in terms of transmissibility of the vertical and angular displacement is taken as criterion for the assessment of the ride comfort in this thesis.

2.2 Type Of Suspension Systems

Suspension systems have been changed and refined as passenger automobile has developed. Design objectives differ between luxury sedans, performance vehicles, small compact vehicles, and light trucks. Design changes for different engine positions, engine mountings and drive. Here, we discuss different suspension system commonly used in passenger cars.

2.2.1 Front Suspension

Front suspension designs have been developed from relatively rugged solid-axle designs to the modern lightweight, high-strength, strut-type independent designs. These have been upgraded with added linkage.

Dependent-type, front-suspension systems with solid axles were used on early domestic vehicles [10]. On a dependent suspension the motion at one wheel causes movement at other wheel. The solid-axle front suspension is used on vehicles that go over rough roads at low speeds. It is the strongest type of the front suspension system (Figure 2.1).

The *independent front suspension* system grew from the demand for improved ride quality and improved handling at increased speeds. In independent suspensions, motion of one wheel does not cause movement at the other wheel. The independent front suspension primarily used the *long-short arm* design (Figure 2.2). Independent front suspension designs on some vehicles took the form of either a leading or a trailing arm design. Today, an increased number of vehicles uses a strut-type of design (Figure 2.3). The strut design provides good ride, good handling, and increased underhood space.

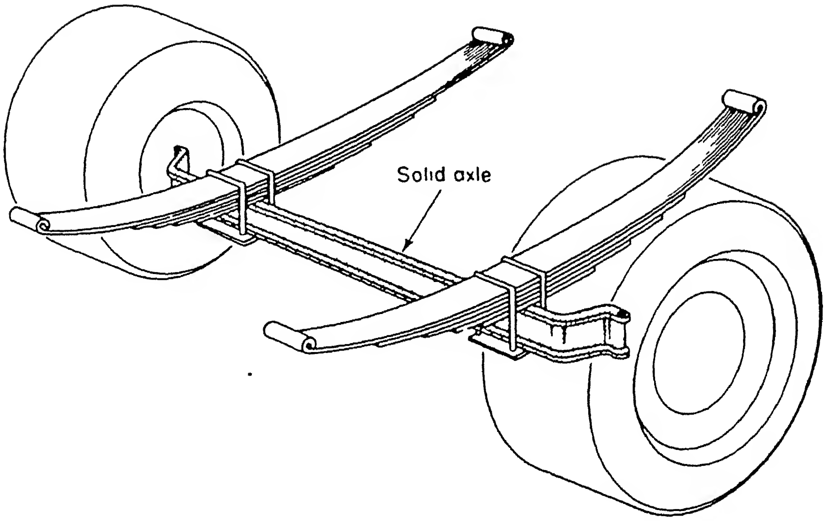


Figure 2.1: Typical Dependent Front Suspension (taken from [10])

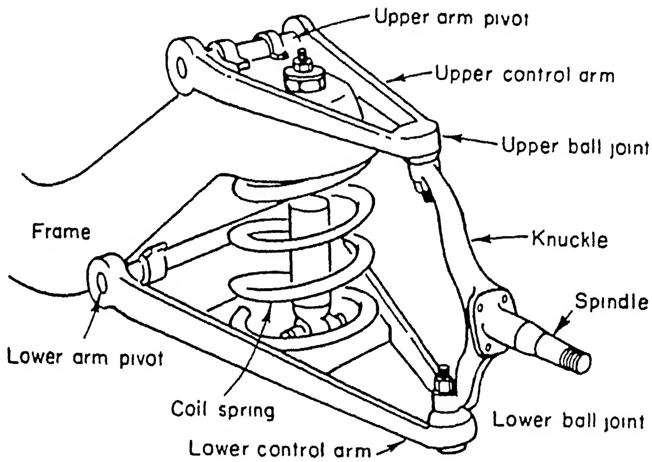


Figure 2.2: Long-short Arm Type Independent Front Suspension (taken from [10])

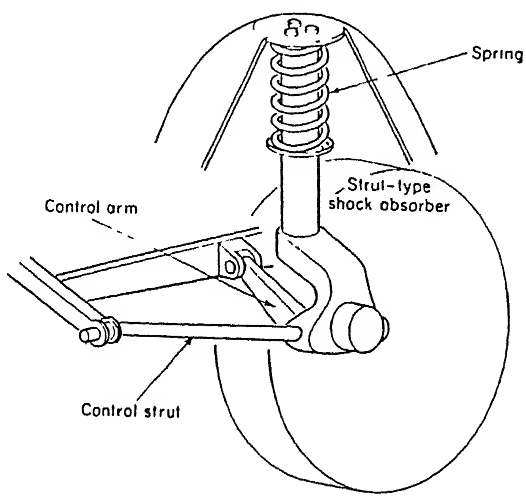


Figure 2.3: Strut Type Front Suspension (taken from [10])

We will consider independent type of front suspension for modeling the suspension system , as this type of suspension is applied in most of the cars designed for comfort ride.

2.2.2 Rear Suspension

Most front-engine, rear-wheel-drive vehicles use a simple dependent rear suspension. Rear-wheel-drive independent suspension is much more complex and expensive.

The move to front-engine, front-wheel-drive has allowed more flexibility in rear suspension design. By moving the drive train to the front, only ride control and breaking reactions are to be controlled by the rear suspension. This has led to the use of simplified dependent suspensions (Figure 2.4), semi-independent suspension (Figure 2.5), and independent rear suspension (Figure 2.6) .

Independent type of rear suspension system is used in next sections for the modeling of the suspension system.

2.3 Suspension Modeling

At the most basic level, all vehicles share the ride isolation properties common to a sprung mass supported by primary suspension systems at each wheel. The essential dynamics can be represented by a quarter car model (Figure 2.7). It consists of a sprung mass supported on a primary suspension, which is in turn connected to the unsprung mass of the axle. The suspension has the stiffness and damping properties. The tyre is represented as a simple spring.

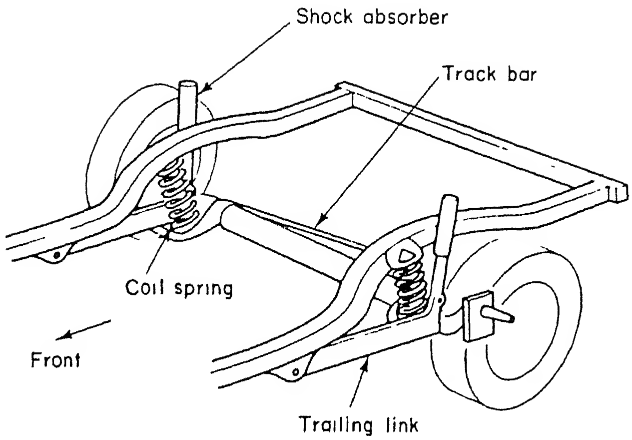


figure 2.4 : Typical Dependent Rear suspension (taken from [10])

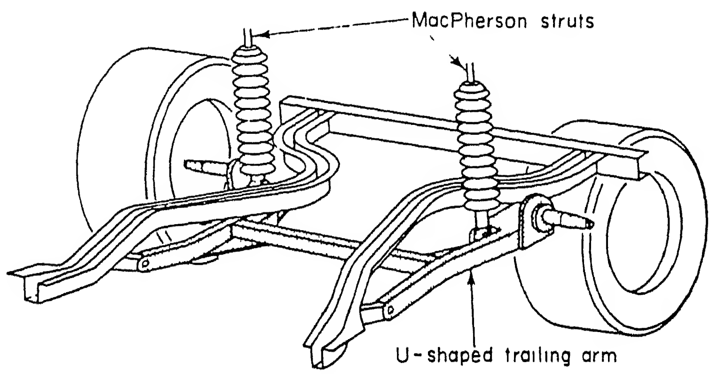


Figure 2.5: Typical Semi-independent Rear Suspension (taken from [10])

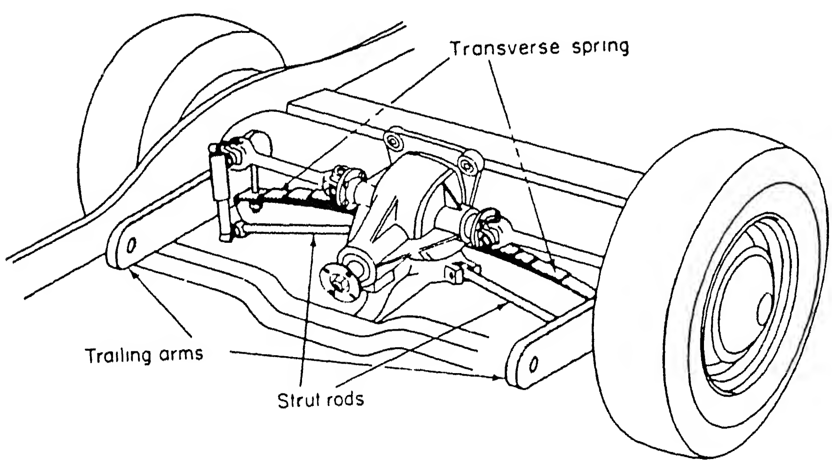


Figure 2.6 :Typical Independent Rear Suspension (taken from [10])

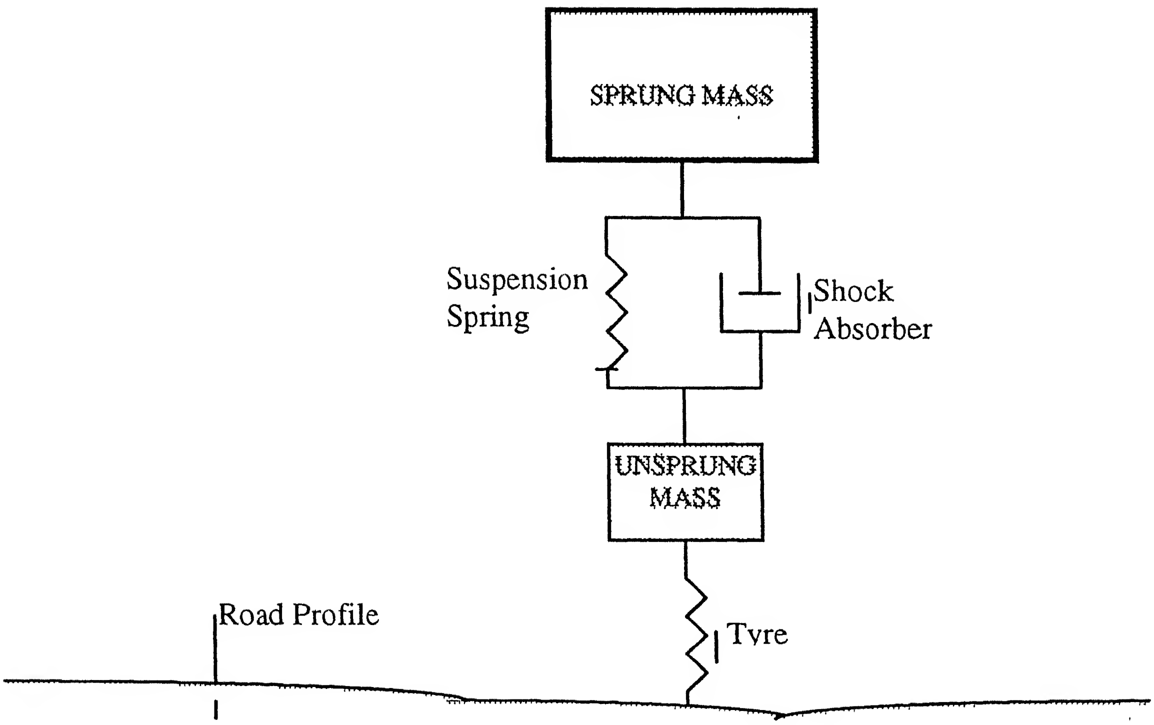


Figure 2.7 Quarter Car Model

2.3.1 Two-dimensional Model

The simple mechanics of quarter car model do not fully represent the rigid body motions that may occur on a car. Because of the longitudinal distance between axles, it is a multi-input system that responds with pitching motion as well as with a vertical bounce. Depending on the road and speed conditions, one or the other motions may be largely absent. A two-dimensional dynamic response is essential for the analysis of this motion. For this, the suspension system can be modeled as shown in Figure 2.8. It consists of a sprung mass supported on the two axles by means of primary suspension, that is, a suspension coil spring and a shock absorber (damper). Each axle contains a unsprung mass that is supported only by the tyres.

To define the model completely the following parameters are to be specified:

Sprung mass (M_s)

Front and rear unsprung mass(M_{fu}, M_{ru})

Front and rear coil stiffness (K_{fs}, K_{rs})

front and rear damper coefficient (α_f, α_r)

Front and rear tyre stiffness (K_{ft}, K_{rt})

Axle to axle distance (L)

Polar moment of inertia of the sprung mass (J)

The two-dimensional model of the car suspension system has four degrees of freedom, namely

- Vertical motion of front unsprung mass(q_1)
- Vertical motion of sprung mass(q_2)
- Angular motion of sprung mass(q_3)
- Vertical motion of rear unsprung mass(q_4)

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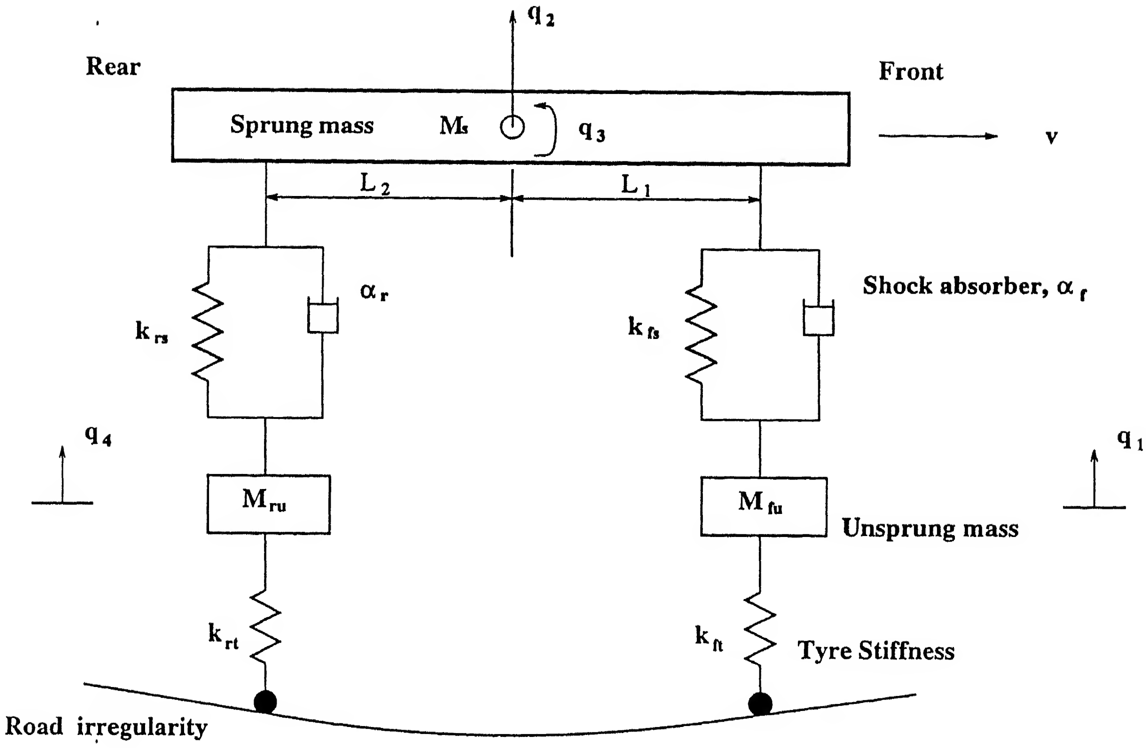


Figure 2.8 Two-dimensional model of the car suspension

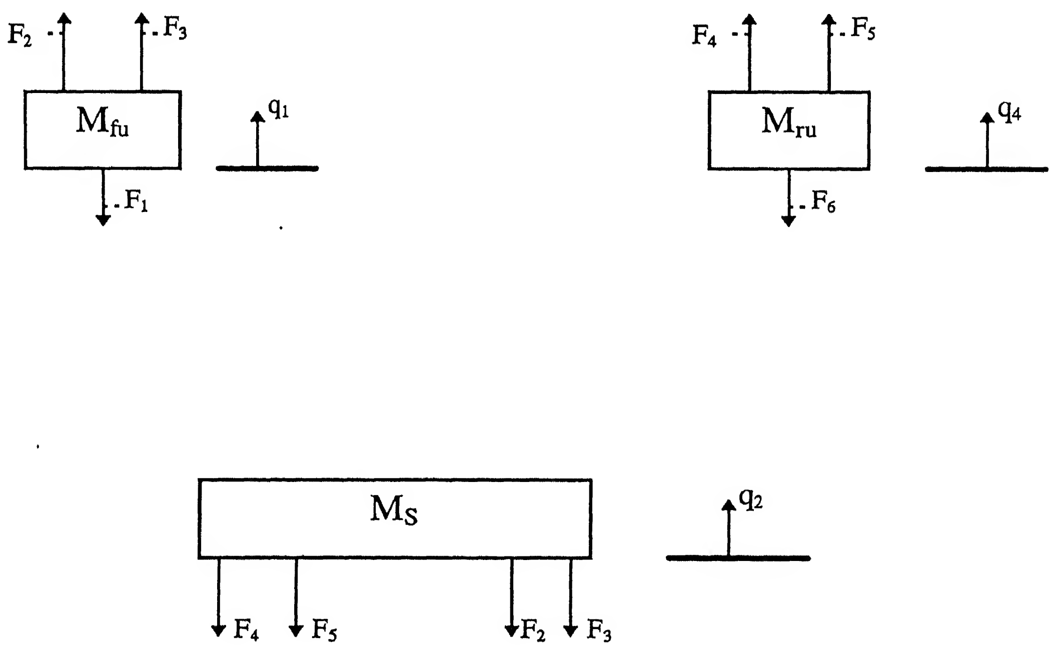


Figure 2.9 Freebody diagram of front and rear unsprung masses and sprung mass

For these four degrees of freedom, the governing differential equations can be written as [7]:

For vertical motion of front unsprung mass

$$\frac{d^2q_1}{dt^2}=\frac{(F_2+F_3-F_1)}{M_{fu}} \tag{2.1}$$

For vertical motion of sprung mass

$$\frac{d^2q_2}{dt^2}=\frac{-(F_2+F_3+F_4+F_5)}{M_S} \tag{2.2}$$

For angular motion (pitching) of sprung mass

$$\frac{d^2q_3}{dt^2}=\frac{(F_4+F_5)L_2-(F_2+F_3)L_1}{J} \tag{2.3}$$

For the vertical motion of rear unsprung mass

$$\frac{d^2q_4}{dt^2}=\frac{(F_4+F_5-F_6)}{M_{ru}} \tag{2.4}$$

where the F_1 to F_6 are the forces due to relative deformation in the springs and velocities in damper (Figure 2.9). These forces can be written with d_1 , d_2 , d_3 , d_4, as relative deformations in front tyre, the front spring, the rear tyre and rear spring as,

$$\begin{aligned} &F_1=K_{ft}d_1 && F_2=K_{fs}d_2 \\ &F_3=\alpha_f d_2 && F_4=Krsd_4 \\ &F_5=\alpha_r d_1 && F_6=Krtd_3 \end{aligned} \tag{2.5}$$

Moreover these deformations can be written as follows

$$d_1=q_1-f_1\,(t) \qquad\qquad d_2=q_2+l_1q_3-q_1$$

$$d_3=q_4-f_2\,(t) \qquad\qquad d_4=q_2-l_2q_3-q_4 \tag{2.6}$$

The time varying functions f_1 (t) and f_2 (t) are road irregularities on front and rear tyres as a function of time. As the car traverses a road, these roughness excitations at different axles are not independent. The rear wheel experiences nearly the same input profile as the front wheel, only delayed in time. The time delay is equal to the axle to axle distance divided by the speed of the travel. Solutions of equations (2.1) to (2.4) will give the bouncing and pitching dynamics of the car.

The effect of road irregularities across the width of the car cannot be seen in the two-dimensional model. Also the above model cannot be used to get the dynamics of the car in rolling. Thus, we consider the three-dimensional model.

2.3.2 Three-dimensional Model

The need of complete analysis of the dynamics of the car necessitates the three-dimensional model for the suspension system. Clearly, the three-dimensional model should contain modeling of all four wheels of the car with the sprung mass resting on the two axles with primary suspension on all the four wheels. So, this model can be seen as a mass resting on four spring-damper pairs which are in turn, connected to a spring mass system. Such a model is shown in Figure 2.10.

To define this model, in addition to unsprung masses, spring stiffnesses, damper coefficients, and

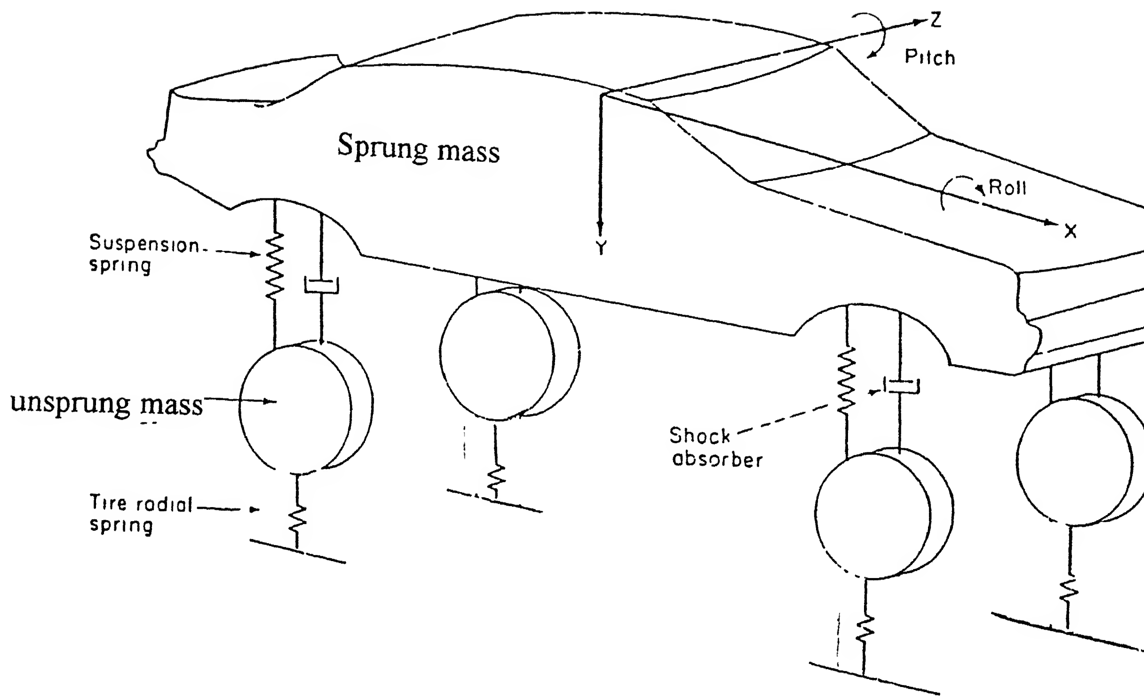


Figure 2.10 : Three-dimensional Model of The Suspension system

tyre stiffnesses at all the four wheels (front right, front left, rear right, rear left) we need the following considerations:

- Polar moment of inertia of sprung mass in pitching (J_{pitch})
- Polar moment of inertia of sprung mass in rolling (J_{roll})
- Wheel base (axle to axle distance) (L)
- Wheel track (distance between the right and left wheels) (W)
- Position of center of gravity of the sprung mass

This model has eight degrees-of-freedoms. Governing differential equations can be written as follows:

For vertical motion of front right unsprung mass

$$\frac{d^2 q_1}{dt^2} = \frac{(F_2 + F_3 - F_1)}{M_{fur}} \tag{2.7}$$

For vertical motion of front left unsprung mass

$$\frac{d^2 q_2}{dt^2} = \frac{(F_5 + F_6 - F_4)}{M_{ful}} \tag{2.8}$$

For vertical motion of rear right unsprung mass

$$\frac{d^2 q_3}{dt^2} = \frac{(F_8 + F_9 - F_7)}{M_{rur}} \tag{2.9}$$

For vertical motion of rear left unsprung mass

$$\frac{d^2 q_4}{dt^2} = \frac{(F_{11} + F_{12} - F_{10})}{M_{rul}} \tag{2.10}$$

For vertical motion of sprung mass

$$\frac{d^2 q_5}{dt^2} = \frac{-(F_2 + F_3 + F_5 + F_6 + F_8 + F_9 + F_{11} + F_{12})}{M_S} \tag{2.11}$$

For pitching motion of sprung mass

$$\frac{d^2 q_6}{dt^2} = \frac{(F_2 + F_3)L_1 + (F_5 + F_6)L_1 - (F_8 + F_9)L_2 - (F_{11} + F_{12})L_2}{J_{pitch}} \tag{2.12}$$

For rolling motion of sprung mass

$$\frac{d^2 q_7}{dt^2} = \frac{(F_2 + F_3)L_3 - (F_5 + F_6)L_4 + (F_8 + F_9)L_3 - (F_{11} + F_{12})L_4}{J_{roll}} \tag{2.13}$$

The springs and dampers have no lateral resistance and as no lateral force on the model is considered. The yawing motion, the rotation about vertical axis has been ignored, in the above three-dimensional model.

Hence, equations (2.7) to (2.13) describes the dynamics of the model in bouncing, pitching and rolling. By imparting different excitation functions ($f_1(t)$ to $f_4(t)$) to four tyres, any road excitation can be simulated.

The forces and relative deformation in the system are given by

$F_1 = K_{ftr}\, d_1$ $\quad$ $F_2 = K_{fsr}\, d_2$ $\quad$ $F_3 = \alpha_{fsr}\, \dot{d}_2$

$F_4 = K_{ftl}\, d_3$ $\quad$ $F_5 = K_{fsl}\, d_4$ $\quad$ $F_6 = \alpha_{fsl}\, \dot{d}_4$ $\quad$ (2.14)

$F_7 = K_{rtr}\, d_5$ $\quad$ $F_8 = K_{rsr}\, d_6$ $\quad$ $F_9 = \alpha_{rsr}\, d_6$

$F_{10} = K_{rtl}\, d_7$ $\quad$ $F_{11} = K_{rsl}\, d_8$ $\quad$ $F_{12} = \alpha_{rsl}\, d_8$

$$
\begin{aligned}
d_1 &= q_1 - f_1(t) & d_2 &= q_5 - q_6\, l_1 - q_7\, l_3 - q_1 \\
d_3 &= q_2 - f_2(t) & d_4 &= q_5 - q_6\, l_1 - q_7\, l_4 - q_2 \\
d_5 &= q_3 - f_3(t) & d_6 &= q_5 + q_6\, l_2 - q_7\, l_3 - q_3 \\
d_7 &= q_4 - f_4(t) & d_8 &= q_5 + q_6\, l_2 + q_7\, l_4 - q_4
\end{aligned}
\tag{2.15}
$$

To calculate the dynamic response of the car, the steps are as follows:

1. Derive the excitation functions $f_1(t)$ to $f_4(t)$ best representing the road conditions to be simulated.
2. Calculate the relative displacements of all the points in the suspension system by using equation (2.15).
3. Calculate the resistive forces in all the flexible elements of the suspension system using equation (2.15) in equation (2.14).
4. These forces can be substituted in equations (2.7) to (2.13) to get the governing equations of the suspension system.
5. These equations now can be solved for finding the dynamic response of the car to the excitation given by the road in form of excitation functions $f_1(t)$ to $f_4(t)$.

2.4 Summary

The primary function of the suspension system is to keep the tyres on the road surface and to provide a smooth ride for driver and the passengers. The suspension is designed to absorb and dampen the vibration due to road roughness before they reach the passenger compartment. The road roughness profile can be represented as the sum of sine waves varying in amplitude, frequency and phase. The quality of the ride is a subjective perception. We are using the transmissibility of the vibrations as the index of the ride quality. To improve open ride quality and/or handling properties of the vehicle, different types of the suspension systems starting from dependent front suspension and dependent rear suspension to strut type of front suspension and independent rear suspensions are in use with many variations among them. In this thesis, we are assuming a car with both front and rear suspension as of independent type.

The quarter-car model of the suspension system does not give much information about the dynamics of the car. The two-dimensional model developed describes the motion of the car in bouncing and pitching but it fails in providing the dynamics of the car in rolling motion. This shortcoming of the two-dimensional model necessitated the development of three-dimensional model of the car. The three-dimensional model of the car has seven degrees of freedom and has 21 parameters. Equations (2.7) to (2.13) describe the complete motion of three-dimensional model. With the help of this three-dimensional model, motion of sprung mass in bouncing, pitching and rolling can be analyzed. The road irregularities as a functions of time are used as excitation function for the model.

In the next chapter, we present a overview of genetic algorithm with a brief discussion of its applications in engineering problems.

Chapter 3

Genetic Algorithms : An Overview

Man has been always trying to mimic the nature. He makes use of principles of nature in order to make his living better. Genetic algorithm is one such example [12,14]. Towards a step in mimicking the evolution, the effort in genetic algorithm has been to find solutions through computers . This chapter gives a bird's eye view of the technique of genetic algorithm.

3.1 Genetic Algorithm : Genetics and Algorithm

Once there was a monkey and after few eras there developed a man. This is an example of evolution. Natural evolution involves perseverance and inheritance of good characteristics and simultaneous creation of new and better ones. On the most fundamental level all these are

achieved through evolution of thread-like structure called chromosomes. The perseverance comes through selection and creation comes through mixing and recombination of genetic material in the chromosomes.

In a genetic algorithm (GA), all these are achieved through [12]:

- Selection
- Crossover, and
- Mutation

To perform these operations a GA need a population. The population has a number of members which are called individuals in GA terminology. Individuals contains variables coded like genes in chromosomes. In a simple GA, variables are coded in the form of strings containing alleles (either 0 or 1).

3.1.1 Selection

On the population of individuals, usually the first operation is selection. Each individual, depending open its alleles have some decoded value and an associated fitness. On the lines of Darwin's principle of *survival of the fittest,* this fitness acts as the criterion for the selection of an individual for taking part in crossover. The individual with higher fitness is selected more often then the one with lower fitness. Selected individuals are kept in a 'mating pool'. In the mating pool, the individuals participate in creating new children points.

For selection a number of schemes are developed, but the basic idea behind all of them is to pick above-average individuals for filling the mating pool with their multiple copies in some probabilistic manner.

One of the methods of selection is the *stochastic remainder proportionate selection.* In maximization problems, using this method, probability of selection of an individual is $f_i / \Sigma f_i$, where f_i is the fitness value of the i-th individual and Σf_i is the sum of the fitness of all the individuals in the population. Obviously an individual with large f_i possesses higher chances of selection. This method is used for maximization problems. For minimization problems fitness function is to modified suitably [6,12].

Another most commonly used selection method is the *tournament selection* [6,12]. The advantage of this method is that it can be applied to both minimization and maximization problems without modifying the fitness function. In this methods "s" number of individuals are compared based on their fitness. For maximization problem the individual having highest fitness is said to be winner and selected to go to the mating pool while in minimization problems one with the lowest fitness value is said to be the winner and goes to the mating pool. The parameter, "s" is the number of individuals compared at a time and is called "tournament size". Generally, the tournament size is kept as 2.

3.1.2 Crossover

> *"The remarkable ability of genetic algorithms to focus their attention on the most promising part of a solution space is a direct outcome of their ability to combine strings containing partial solutions".* [Holland J.H. 'Genetic Algorithms' Scientific America July 1992]

After creating the mating pool that is after selection, the second operator in GA is crossover. In nature, when sperm and ova fuse, mating chromosomes mix and exchange the genetic information. Similarly, in crossover operator. strings in the mating pool exchange information to

create new strings. Mating individuals are called parent points or parent individuals. In a GA two parent strings are picked up at random from the mating pool and across the length of the string a random cross site is chosen. To exchange the information, alleles on one site of the cross site are swept between the two individuals to create two new strings called "children points". For example, if s_1 and s_2 two individuals are mating with a random cross site as shown

$$s_1 = 1\ 1\ 0 \mid 0\ 1\ 1$$

$$s_2 = 1\ 0\ 1 \mid 1\ 0\ 0$$

then after crossover the two children points s'_1 and s'_2 will be

$$s'_1 = 1\ 1\ 0\ 1\ 0\ 0$$

$$s'_2 = 1\ 0\ 1\ 0\ 1\ 1$$

The crossover is the operator which combines the genetic information of two solutions. The crossover is not done to all population members, instead it is done with a probability of p_c .

The above described crossover is called a single point crossover. There are many other cross over operators available, such as multi-point crossover, uniform crossover [12]. Many problem specific crossover operators also available [12].

3.1.3 Mutation

After performing the crossover the new population is almost ready. In order to avoid premature convergence a mutation is used. As discussed in [12] "mutation is insurance against premature convergence". This operator, "mutation" changes alleles (0 changing to 1 or vise versa) occasionally with a probability p_m.

The mutation works as shown below:

1 0 0 $\boxed{1}$ 0 1 —mutation→ 1 0 0 0 0 1

The flow chart of a simple genetic algorithm is shown in Figure 3.1.

Mutation may provide a feature to the child string which none of its parents had. After doing mutation the new population is complete and a generation of GA is over.

3.2 Applications Of Genetic Algorithms

GA works through a coding of variables instead of variables themselves. So, GA works with a search space which is discretized, regardless of the function's continuity. This increased the range of engineering problems that can be solved by GAs. With this the population-based approach of GAs enables the parallel processing of many solutions in different regions of the search space. Along with these advantages the fact that GAs requires only the fitness information from the objective function makes the GAs more robust then any other search and optimization algorithm. With these and many more advantages GAs had been successfully applied to a breathtaking range of problems, from design of mechanical components [8], structure designs [4] , metallurgical problems [18] and even to problems like criminal face recognition [2] . GAs are also used for solving optical filter design [5] and ambiguous shape modeling [1] problems. The list of application of GAs is growing very fast, GAs are getting more and more advanced with development of new ways of coding, new selection operators, new crossover operators and many other related developments. In this thesis, we use GAs to find optimal car suspension system for different objectives of a practical design.

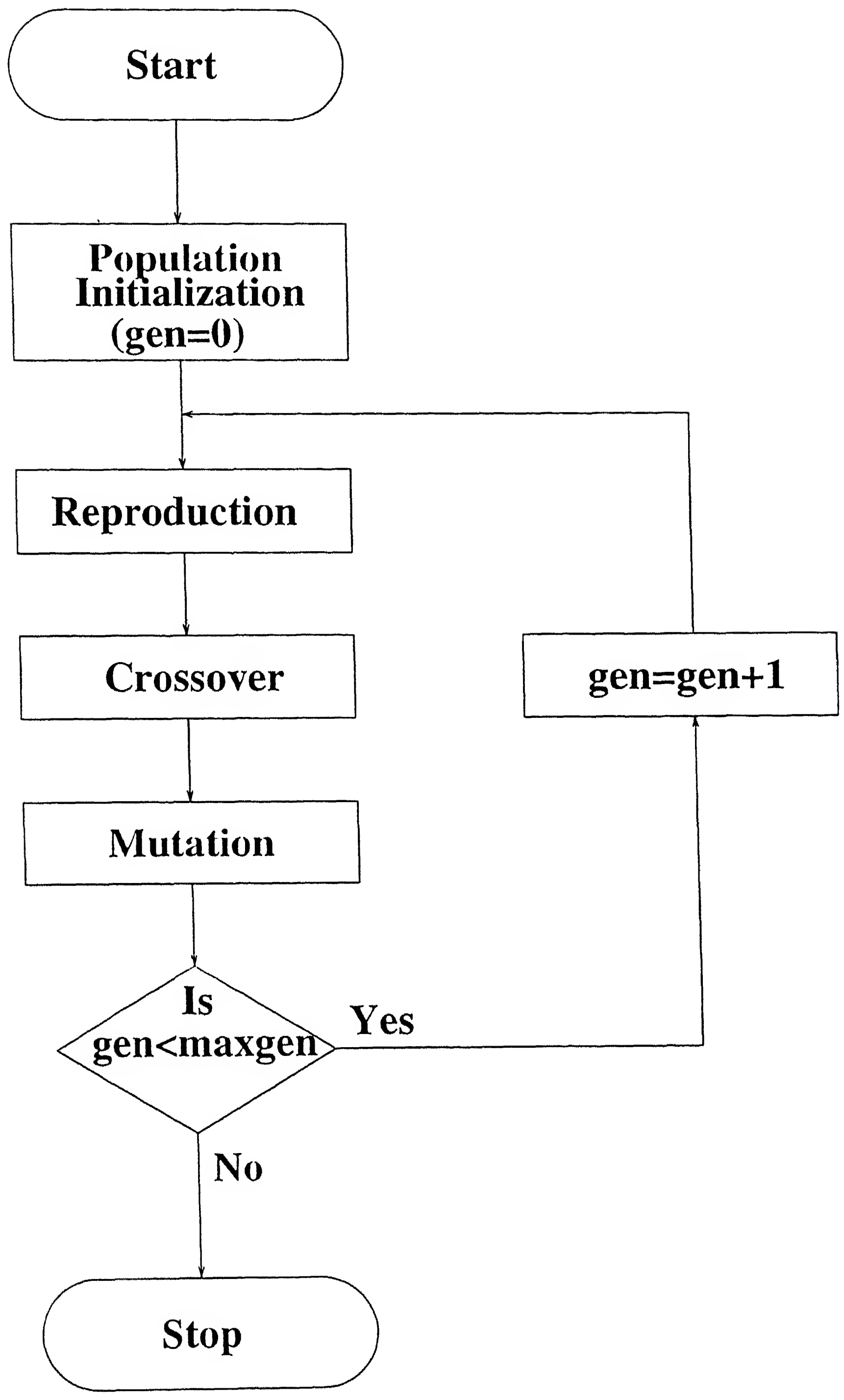


Figure 3.1: Flow Chart of a simple genetic algorithm

3.3 Summary

The search procedure based on the mechanics of the natural genetics and natural selection is the quintessence of genetic algorithms. The variables of the functions are coded in the form of a string. Strings get their fitness values from the underlying objective function. GA has three main operators, namely, *selection, crossover* and *mutation*. Selection is the process of forming the mating pool of successful individuals. In crossover, two individuals are chosen at random and substring on one side of a randomly selected cross site is swapped to create new children points. Mutation tries to introduce some extra information that is usually not possible by the crossover operator. Because of a number of advantages over other search and optimization algorithms, GAs have a wide variety of applications in many engineering fields. In this thesis we use GAs to find the optimal car suspension system.

Chapter 4

Optimal Suspension Design Formulation

In any system design problem, the objective is to first identify a number of salient parameters representing the problem and then to find a suitable set of values of these parameters. Different combinations of these parameters leads to different performances of the system. One may be better than others or some may be worse than other in respect to an objective. Thus there is a need to find that combination of system parameters which give the best performance of the system. One of the ways to achieve this is to pose the problem as an optimization problem.

All possible parameters of a problem are not taken as design variables [7]. In an optimization problem salient parameters are first chosen as design variables. The other variables assumed as constant. Once the variables are identified, the next step is to chose an objective function. The

objective for which the optimization is to be carried out, should be very carefully chosen, because same problem will lead to different solutions for different objectives. The next step is to identify constraints which might arise to satisfy certain restrictions and limitations in behavior of the system. In this chapter, we discuss optimal design of the suspension system of a passenger car by identifying all components of a optimal design formulation, mentioned above.

4.1 Two-dimensional Model

The car suspension design can be achieved by considering two-dimensional and three-dimensional model of the suspension system. In a two-dimensional model the suspension system is modeled with only two wheels, one at front and other at rear. With two-dimensional model the vertical bounce motion and pitching motion of the sprung mass can be analyzed. As discussed in Chapter 3, equation (2.1) to (2.4) govern the dynamics of the four degree-of-freedom two-dimensional model of the suspension system. In the three-dimensional model, in addition to above stated motions of sprung mass, the rolling motion of the sprung mass can also be analyzed. We first discuss the two-dimensional model and then we will discuss the three-dimensional model.

4.1.1 Mathematical Modeling

Analysis of the equations (2.1) to (2.4) describing the dynamics of the two-dimensional model of a car suspension shows that they belong to the group of ordinary, nonlinear, second order differential equations. The solution of these equations in closed form is impossible to find. This is primarily because of the restrictions in spring and damper characteristics followed by the Tata Engineering and Locomotive Company (TELCO), Pune. For example, stiffness of rear spring and damping coefficients of dampers and at front and rear are not constant with respect to the deformation and

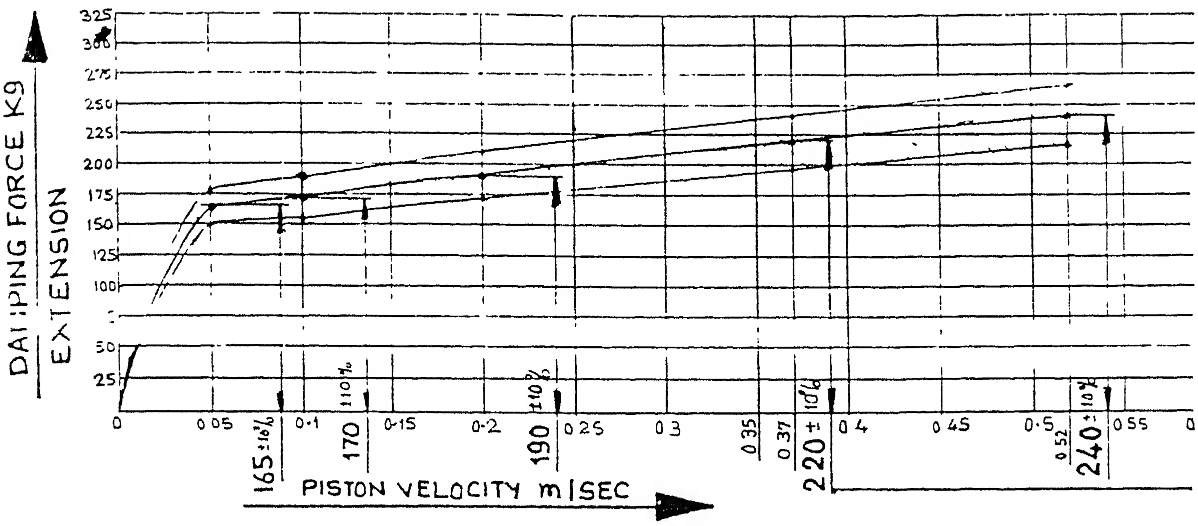


Figure 4.1a: Typical Characteristics of Front Damper (Obtained from TELCO, Pune)

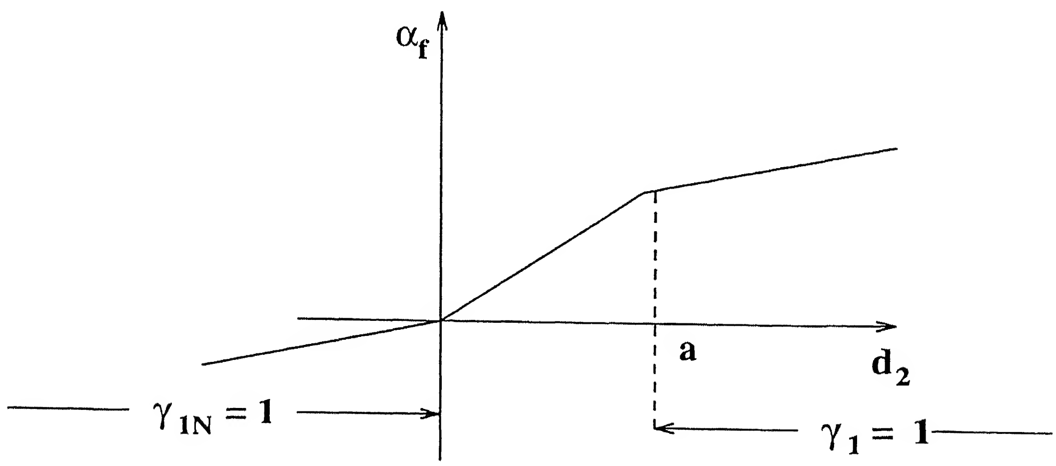


Figure 4.1b: Simple Representation of Damper Characteristics (Front Damper)

velocities which they are subjected to. This makes the coefficients of the equations variables and makes it difficult to find exact solution to differential equations.. Typical characteristics of the front damper is shown in Figure 4.1a. Also, these elements have different resistance in bounce and recoil. To take this in account, the force equations in equation (2.5) are modified as follows:

$$F_3 = [\alpha_{f1} + \gamma_{1N}(\alpha_{f3} - \alpha_{f1}) + \gamma_1(1-\gamma_{1N})(\alpha_{f2} - \alpha_{f1})]\dot{d}_2 - \gamma_1(1-\gamma_{1N})(\alpha_{f2} - \alpha_{f1})a \qquad (4.1)$$

$$F_4 = [K_{rs1} + \gamma_{2N}(K_{rs3} - K_{rs1}) + \gamma_2(1-\gamma_{2N})(K_{rs2} - K_{rs1})]d_4 - \gamma_2(1-\gamma_{2N})(K_{rs2} - K_{rs1})b \qquad (4.2)$$

$$F_6 = [\alpha_{r1} + \gamma_{3N}(\alpha_{r3} - \alpha_{r1}) + \gamma_3(1-\gamma_{3N})(\alpha_{r2} - \alpha_{r1})]\dot{d}_4 - \gamma_3(1-\gamma_{3N})(\alpha_{r2} - \alpha_{r1})c \qquad (4.3)$$

where γ_1 is 1 for $d_2 \geq a$ and zero elsewhere end γ_{1N} is one for $d_2 < 0$ and zero elsewhere. Similarly γ_2 and γ_{2N} are defined for d_4 and γ_3 and γ_{3N} are defined for $\dot{d}_4$.

If the Z_f, Y_f, Z_r, Y_r, Z_{rs} and Y_{rs} are chosen such that

$$F_3 = Z_f\dot{d}_2 - Y_f a$$

$$F_4 = Z_{rs}d_4 - Y_{rs}b$$

$$F_6 = Z_r\dot{d}_4 - Y_r c$$

then to get the solution, the equations (2.1) to (2.4) can be transformed into a system of first- order differential equations. After the transformation they can be written as follows:

$$\dot{Q} = K_M Q + C \qquad (4.4)$$

Where

$$Q = \begin{bmatrix} \dot{q}\frac{M_{fu}}{g} & \dot{q}\frac{M_s}{g} & \dot{q}\frac{J}{g} & \dot{q}\frac{M_{ru}}{g} & q_1 & q_2 & \dot{q}_3 & q_4 \end{bmatrix}^T$$

$$Q = \begin{bmatrix} \dot{q}_1 & \dot{q}_2 & \dot{q}_3 & \dot{q}_4 & q_1 & q_2 & q_3 & q_4 \end{bmatrix}^T$$

$$K_M = \begin{bmatrix} -Z_f & Z_f & l_1 Z_f & 0 & -K_{fs} - K_{ft} & K_{fs} & K_{fs} l_1 & 0 \\ Z_f & -Z_f - Z_r & -l_1 Z_f + l_2 Z_r & Z_r & K_{fs} & -K_{fs} - Z_{rs} & -K_{fs} l_1 + Z_{rs} l_2 & Z_{rs} \\ l_1 Z_f & l_2 Z_r - l_1 Z_f & -l_2{}^2 Z_r - l_1{}^2 Z_f & -l_2 Z_r & l_1 K_{fs} & l_2 Z_{rs} - l_1 K_{fs} & -l_2{}^2 Z_{rs} - l_1{}^2 K_{fs} & -l_2 Z_{rs} \\ 0 & Z_f & -l_2 Z_r & -Z_r & 0 & Z_{rs} & -l_2 Z_{rs} & -Z_{rs} - K_{rt} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} K_{ft} - f_1(t) - aY_f, & aY_f + bY_{rs} + cY_r, & -bl_2 Y_{rs} - cY_r + al_1 Y_f, & -bY_{rs} - cY_r + K_{rt} f_2(t), & 0, & 0, & 0, & 0 \end{bmatrix}^T$$

The equations can now be solved by using a numerical integrator such as Runga-Kutta method. The solution vector will contain eight elements corresponding to these eight rows of above matrix. Let the solution vector be { Q_1,Q_2, Q_3, Q_4, Q_5, Q_6, Q_7, Q_8 }^{T}, where Q_1,Q_2 and Q_4 represent the vertical velocities of front unsprung mass, sprung mass and rear unsprung mass respectively. Q_3 represents the pitching velocity of the sprung mass. Q_5, Q_6 and Q_8 represent the vertical displacements of front

unsprung mass, sprung mass and rear unsprung mass respectively. Q_7 represents the pitching amplitude of the sprung mass.

4.1.2 Design Variables

The two-dimensional model of the car suspension system given in Chapter 2 has eleven parameters in its description. But all of these are not chosen as design variables, some of them are really can not be the design variables. In order to make an optimization procedure converge in a reasonable time and keeping in view of their critical role in dynamic behavior of the system, following parameters are chosen as design variables:

- Front spring stiffness (K_{fs})
- Front damper coefficient (α_{f1})
- Rear spring stiffness (K_{rs1})
- Rear damper coefficient (α_{r1})

In the previous section, it has been seen that, except front spring stiffness, all have different values in different ranges of displacements and velocities they are subjected to. Specifically, they can take three different values. To reduce the number of design variables, only one of the stiffness value and only one damping coefficient value for each damper is used as variable and the other two are assumed as linearly proportional to the variable. These two prepositional ratios will be fixed on the basis of the characteristics given by the ERC division of TELCO, Pune.

4.1.3 Objective Function

In the simplest sense, the objective function is a mathematical relationship among design variables formulating the objective which is to be optimized. In this problem, a set of design variables gives a set of differential equations to solve and the solution of the equations gives the response of the system. From this response the objective is to be derived. The objective depends on the underlying road excitation. If the car is going over a bump, the shape of bump will act as road irregularity functions that are $f_1(t)$ and $f_2(t)$ in the formulation. The bump can be modeled as a half sine curve Then $f_1(t)$ can be written as

$$f_1(t) = h\sin(2\pi vt / l) \qquad 0 \leq t \leq l/v$$

Where

h = height of the bump.

v = velocity of the car.

l = breadth of the bump.

t= time of travel.

Rear wheel also experiences the same bump after time L/v , hence

$$f_2(t) = h\sin(2\pi v / l(t - L / v)) \qquad L/v \leq t \leq (L+l)/v$$

with L as the distance between rear and front axles.

As the car goes over the bump, the sprung mass also goes up, but after coming back on level road sprung mass keeps on moving up and down depending on the underlying natural frequency of the car. This motion of the sprung mass gives a feeling of discomfort to the passenger. The amplitude of

this subsequent motion of the sprung mass should be minimized. In other words, the effect of bump transferred to sprung mass, that is the transmissibility of the displacement should be minimum. The transmissibility is used as the objective function for the two-dimensional model. Mathematically, the objective function is as follows:

$$\text{Minimize} \quad \max(Q_6)\,/h \tag{4.5}$$

Since the sprung mass will follow the bump, during its motion over the bump, the transmissibility is defined after the respective wheel have passed the bump.

4.1.4 Constraints

Constraints are the criteria for a rejection or acceptance of a solution during the optimization procedure. The constraints thus play an important role in optimization procedure.

The discomfort of passengers due to vertical motion of sprung mass does not end with its amplitude of the motion. The jerk, that is rate of change of vertical acceleration, experienced by the passenger also gives much discomfort to the passengers. So, for having comfortable ride, the jerk should be within prescribed limits. This constitutes one constraint for the car suspension design problem . It is a common practice in automobile industries to limit the jerk at $18 m/s^3$ [19]. Thus mathematically

$$\dddot{Q}_6 \leq 18 \text{ m/s}^3$$

$$\text{or,} \qquad 18 - \dddot{Q}_6 \geq 0 \tag{4.6}$$

Complete NLP problem can now be stated as follows:

$$\text{Minimize} \ \max(Q_6)/h$$

Subjected to

$$18 - \dddot{Q}_6 \geq 0$$

$$0 \leq K_{fs} \leq 5.0$$

$$0 \leq K_{rs1}, \leq 5.0$$

$$0 \leq \alpha_{f1} \leq 5.0$$

$$0 \leq \alpha_{r1} \leq 5.0$$

4.2 Three-dimensional model

Three-dimensional model of the suspension system of a car enables us to include the rolling motion of the car. Effect of undulations of the road across its length and along its breadth both can be taken care of with the three-dimensional model. The seven degrees-of -freedom model (Figure 2.10), gives the description of the bouncing motion, pitching motion and rolling motion of the sprung mass. Motion of unsprung masses are also described by the model. Here we discuss this three-dimensional model of the suspension system.

4.2.1 Mathematical model

The three-dimensional model and its governing equations can be analyzed along similar lines to the above two-dimensional model. Differential equations are of the same nature and so are the flexible element characteristics. Mathematical complexity is increased in terms of number of equations and

number of terms in the equations. Due to nonlinear characteristics of the dampers and rear springs force equations in equation (2.15) should be modified as the following:

$$F_3 = [\alpha_{fr1} + \gamma_{1N}(\alpha_{fr3} - \alpha_{fr1}) + \gamma_1(1-\gamma_{1N})(\alpha_{fr2} - \alpha_{fr1})]\dot{d}_2 - \gamma_1(1-\gamma_{1N})(\alpha_{fr2} - \alpha_{fr1})a_r \quad (4.7)$$

$$F_6 = [\alpha_{fl1} + \gamma_{2N}(\alpha_{fl3} - \alpha_{fl1}) + \gamma_2(1-\gamma_{2N})(\alpha_{fl2} - \alpha_{fl1})]\dot{d}_4 - \gamma_2(1-\gamma_{2N})(\alpha_{fl2} - \alpha_{fl1})a_l \quad (4.8)$$

$$F_8 = [K_{rsr1} + \gamma_{3N}(K_{rsr3} - K_{rsr1}) + \gamma_3(1-\gamma_{3N})(K_{rsr2} - K_{rsr1})]d_6 - \gamma_3(1-\gamma_{3N})(K_{rsr2} - K_{rsr1})b_r \quad (4.9)$$

$$F_9 = [\alpha_{rr1} + \gamma_{4N}(\alpha_{rr3} - \alpha_{rr1}) + \gamma_4(1-\gamma_{4N})(\alpha_{rr2} - \alpha_{rr1})]\dot{d}_6 - \gamma_4(1-\gamma_{4N})(\alpha_{rr2} - \alpha_{rr1})c_r \quad (4.10)$$

$$F_{11} = [K_{rsl1} + \gamma_{5N}(K_{rsl3} - K_{rsl1}) + \gamma_5(1-\gamma_{5N})(K_{rsl2} - K_{rsl1})]d_8 - \gamma_5(1-\gamma_{5N})(K_{rsl2} - K_{rsl1})b_l \quad (4.11)$$

$$F_{12} = [\alpha_{rl1} + \gamma_{6N}(\alpha_{rl3} - \alpha_{rl1}) + \gamma_6(1-\gamma_{6N})(\alpha_{rl2} - \alpha_{rl1})]\dot{d}_8 - \gamma_6(1-\gamma_{6N})(\alpha_{rl2} - \alpha_{rl1})c_l \quad (4.12)$$

Again we choose Z_{fr}, Z_{fl}, Y_{fr}, Y_{fl}, Z_{rr}, Y_{rr}, Z_{rl}, Y_{rl}, Z_{rsr}, Y_{rsr}, Z_{rsl} and Y_{rsl} such that

$F_3 = Z_{fr}\,\dot{d}_2 - Y_{fr}\,a_r$ $\qquad$ $F_6 = Z_{fl}\,\dot{d}_4 - Y_{fl}\,a_l$

$F_8 = Z_{rsr}\,d_6 - Yr_{sr}\,b_r$ $\qquad$ $F_{11} = Z_{rsl}\,d_8 - Y_{rsl}\,b_l$

$F_9 = Z_{rr}\,\dot{d}_6 - Y_{rr}\,c_r$ $\qquad$ $F_{12} = Z_{rl}\,d_8 - Y_{rl}\,c_l$

Transformation of equations (2.7) to (2.13) into a first order equation system leads to following matrix equation:

$$\dot{Q} = K_M Q + C \quad (4.14)$$

Where

$$
K_M = \begin{bmatrix}
-Z_{fr} & 0 & 0 & 0 & Z_{fr} & -Z_{fr}l_1 & -Z_{fr}l_3 & -K_{fsr}-K_{ftr} & 0 & 0 & 0 & K_{fsr} & -K_{fsr}l_1 & -K_{fsr}l_3 \\
0 & -Z_{fl} & 0 & 0 & Z_{fl} & -Z_{fl}l_1 & Z_{fl}l_4 & 0 & -K_{fsr}-K_{ftr} & 0 & 0 & K_{fsl} & -K_{fsl}l_1 & K_{fsl}l_4 \\
0 & 0 & -Z_{rr} & 0 & Z_{rr} & Z_{rr}l_2 & -Z_{rr}l_3 & 0 & 0 & -Z_{rsr}-K_{rtr} & 0 & Z_{rsr} & Z_{rsr}l_2 & -Z_{rsr}l_3 \\
0 & 0 & 0 & -Z_{rl} & Z_{rl} & Z_{rl}l_2 & Z_{rl}l_4 & 0 & 0 & 0 & -Z_{rsl}-K_{rtl} & Z_{rsl} & Z_{rsl}l_2 & Z_{rsl}l_4 \\
Z_{fr} & Z_{fl} & Z_{rr} & Z_{fl} & \begin{array}{c}(Z_{fr}\\+Z_{fl}\\+Z_{rr}\\+Z_{rl})\end{array} & \begin{array}{c}(Z_{fr}l_1\\+Z_{rr}l_1\\-Z_{rr}l_2\\-Z_{rl}l_2)\end{array} & \begin{array}{c}(Z_{fr}l_3\\-Z_{fl}l_4\\+Z_{rr}l\\{}_3-Z_{rl}l_4)\end{array} & K_{fsr} & K_{fsl} & Z_{rsr} & Z_{rsl} & \begin{array}{c}-(K_{fsr}\\+K_{fsl}\\+Z_{rsr}\\+Z_{rsl})\end{array} & \begin{array}{c}(K_{fsr}l_1\\+K_{fsl}l_1\\-Z_{rsr}l_2\\-Z_{rsl}l_2)\end{array} & \begin{array}{c}-(K_{fsr}l_3\\-K_{fsl}l_4\\+Z_{rsr}l_3\\-Z_{rsl}l_4)\end{array} \\
-Z_{fr}l_1 & -Z_{fl}l_1 & Z_{rr}l_2 & Z_{rl}l_2 & \begin{array}{c}(Z_{fr}l_1\\+Z_{fl}l_1\\-Z_{rr}l_2\\-Z_{rl}l_2)\end{array} & \begin{array}{c}-(Z_{fr}l_1^2\\+Z_{fl}l_1^2\\+Z_{rr}l_2^2\\+Z_{rl}l_2^2)\end{array} & \begin{array}{c}-(Z_{fr}l_1l_3\\-Z_{rr}l_1l_4\\-Z_{rr}l_2l_3\\+Z_{rl}l_2l_4)\end{array} & -K_{fsr}l_1 & -K_{fsl}l_1 & Z_{rsr}l_2 & Z_{rsl}l_2 & \begin{array}{c}(K_{fsr}l_1\\+K_{fsl}l_1\\-Z_{rsr}l_2\\-Z_{rsl}l_2)\end{array} & \begin{array}{c}-(K_{fsr}l_1^2\\+K_{fsl}l_1^2\\+Z_{rsr}l_2^2\\+Z_{rsl}l_2^2)\end{array} & \begin{array}{c}-(K_{fsr}l_1l_3\\-K_{fsl}l_1l_4\\-Z_{rsr}l_2l_3\\+Z_{rsl}l_2l_4)\end{array} \\
-Z_{fr}l_3 & Z_{fl}l_4 & -Z_{rr}l_3 & Z_{rl}l_4 & \begin{array}{c}(Z_{fr}l_3\\+Z_{rr}l_3\\-Z_{fl}l_4\\-Z_{rl}l_4)\end{array} & \begin{array}{c}-(Z_{fr}l_1l_3\\-Z_{fl}l_1l_4\\-Z_{rr}l_2l_3\\+Z_{rl}l_2l_4)\end{array} & \begin{array}{c}-(Z_{fr}l_3^2\\+Z_{fl}l_4^2\\+Z_{rr}l_3^2\\+Z_{rl}l_4^2)\end{array} & -K_{fsr}l_3 & K_{fsl}l_4 & -Z_{rsr}l_3 & Z_{rsl}l_4 & \begin{array}{c}(K_{fsr}l_3\\-K_{fsl}l_4\\+Z_{rsr}l_3\\-Z_{rsl}l_4)\end{array} & \begin{array}{c}-(K_{fsr}l_1l_3\\-K_{fsl}l_1l_4\\-Z_{rsr}l_2l_3\\+Z_{rsl}l_2l_4)\end{array} & \begin{array}{c}-(K_{fsr}l_3^2\\+K_{fsl}l_4^2\\+Z_{rsr}l_3^2\\+Z_{rsl}l_4^2)\end{array} \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$\dot{Q}=\begin{bmatrix}\frac{M_{fur}}{g}\ddot{q}_1 & \frac{M_{ful}}{g}\ddot{q}_2 & \frac{M_{rur}}{g}\ddot{q}_3 & \frac{M_{rul}}{g}\ddot{q}_4 & \frac{M_s}{g}\ddot{q}_5 & \frac{J_{pitch}}{g}\ddot{q}_6 & \frac{J_{roll}}{g}\ddot{q}_7 & \dot{q}_1 & \dot{q}_2 & \dot{q}_3 & \dot{q}_4 & \dot{q}_5 & \dot{q}_6 & \dot{q}_7\end{bmatrix}^T$$

$$Q=\begin{bmatrix}\dot{q}_1 & \dot{q}_2 & \dot{q}_3 & \dot{q}_4 & \dot{q}_5 & \dot{q}_6 & \dot{q}_7 & q_1 & q_2 & q_3 & q_4 & q_5 & q_6 & q_7\end{bmatrix}^T$$

$C=[\, C_1 \;\; C_2 \;\; C_3 \;\; C_4 \;\; C_5 \; C_6 \; C_7 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0\,]$

where

$$C_1 = K_{ftr} - f_1(t) - a_r Y_{fr}$$

$$C_2 = K_{ftl} - f_2(t) - a_l Y_{fl}$$

$$C_3 = K_{rtr} - f_3(t) - c_r Y_{rr} - b_r Y_{rsr}$$

$$C_4 = K_{rtl} - f_4(t) - c_l Y_{rl} - b_l Y_{rsl}$$

$$C_5 = a_r Y_{fr} + a_l Y_{fl} + b_r Y_{rsr} + b_l Y_{rsl} + c_r Y_{rr} + c_l Y_{rl}$$

$$C_6 = -a_l Y_{fl} l_1 + b_l Y_{rsl} l_2 + c_l Y_{rl} l_2 + c_r Y_{rr} l_2 + b_r Y_{rsr} l_2 - a_r Y_{fr} l_1$$

$$C_7 = a_l Y_{fl} l_4 + b_l Y_{rsl} l_4 + c_l Y_{rl} l_4 - c_r Y_{rr} l_3 - b_r Y_{rsr} l_3 - a_r Y_{fr} l_3$$

These equations now can be solved by using Runga-Kutta method. The solution vector will contain fourteen elements corresponding to each row of above matrix. Let the solution vector be $\{ Q_1, Q_2, Q_3, \ldots, Q_{14} \}^T$, Q_1 to Q_7 gives the velocities respective to $\{q_1, q_2 \ldots q_8\}$ and Q_8 to Q_{14} give the $\{q_1, q_2, \quad, q_8)$ values.

4.2.2 Design Variables

Twenty-one different parameters are required to define the three-dimensional model of a car suspension (Figure 2.10). All of them are not taken as design variables. Springs and dampers of the suspension system are the elements used as design variables.

There are four springs and four dampers in the three-dimensional suspension system model. Two are at the front and two are at the rear. Spring and damper on front right side are considered to have exactly the same characteristics as the spring and damper on the front left side. Similarly the spring and damper at the rear left side are considered same as the spring and damper at the rear right side. Thus, in the 3-D case, the design variables are same as in the 2-D case. Explicitly stating, the design variables chosen for the problem are as follows:

i) Front right spring stiffness(K_{fsr})

ii) Front right damper coefficient(α_{fr1})

iii) Rear right spring stiffness(K_{rsr1})

iv) Rear right damper coefficient(α_{rr1})

For variables (ii), (iii) and (iv) values in different ranges will be taken in constant ratio based on that supplied by TELCO, Pune.

4.2.3 Objective Function

In the three-dimensional model, bouncing, pitching and rolling of the sprung mass can be analyzed. So, the problem can be formulated by taking any one of them on prime importance. Therefore three different objective functions are formulated to optimize design for each motion. The three objectives are as follows:

- Minimization of the transmissibility
- Minimization of amplitude in pitching
- Minimization of amplitude in rolling

4.2.3.1 Minimization of Transmissibility

For the purpose of the minimization of transmissibility, car's motion over a speed breaker is simulated. The left and right tyres of the car moving over a speed breaker experiences the same road profile, with only the difference that rear wheel passes the same profile on the road a little late then the front tyre. In this situation, the forcing function $f_1(t)$ and $f_2(t)$ becomes the same. As the speed breaker is modeled as half sine curve, these functions can be written mathematically as follows:

$$f_1(t)=hsin(2\pi vt/l) \qquad 0\leq t \leq l/v$$

$$f_2(t)=f_1(t)$$

and also

$$f_3(t)=hsin(2\pi v/l(t-L/v)) \qquad L/v \leq t \leq (L+l)/v$$

$$f_4(t)=f_3(t)$$

With these forcing functions, the response of the system can be found by solving equation (4.13), and objective function for the purpose can be written as

$$\text{Minimize} \qquad \max(Q_{12})/h \tag{4.14}$$

Since the sprung mass will follow the bump, during its motion over the bump, the transmissibility is defined after the respective wheel have passed the bump.

4.2.3.2 Minimization Of Amplitude In Pitching

When a car traverses over a road having large undulations, high pitching of sprung mass may result. In such cases the objective of the suspension design should be to minimize the pitching amplitude.

To simulate such a road, a sinusoidal road with wave length equal to the wheel base is simulated. Road excitation functions for this type of road can be written as follows:

$$f_1(t)=hsin(\pi vt/l)$$

$$f_2(t)=f_1(t)$$

$$f_3(t)=hsin((\pi v/l)(t-L/2v)$$

$$f_4(t)=f_3(t)$$

Solving the equation (4.14) enables us to write the objective function as

$$\text{Minimize} \ \max(Q_{13}) \tag{4.15}$$

4.2.3.3 Minimization Of Amplitude In Rolling

If the right and left wheels moves over different road conditions, rolling motion will take place. Most severe rolling will be encountered when one wheel is going over a bump and other is going in a hole and vice versa. For getting the objective function, car's travel over a twisting road in simulated. A twisting road in one which has waviness and left and right wheel have out of phase excitation. Excitation functions for such a road can be written as follows:

$$f_1(t) = hsin(\pi vt/l)$$

$$f_2(t) = -f_1(t)$$

$$f_3(t)= hsin((\pi v/l)(t-L/v))$$

$$f_4(t) = -f_3(t)$$

Equation (4.13) now can be solved with these forcing functions. The objective function then is

$$\text{Minimize} \ \max(Q_{14}) \tag{4.16}$$

4.2.4 Constraints

Constraints for the optimization problem come from other related aspects which are not covered in the objective function formulation. In order to make the suspension design more realistic and practically applicable, a few other constraints are induced based on discussions with engineers of TELCO, Pune [19].

i) As discussed in the two-dimensional model, the jerk experienced by the passenger should be within prescribed limits. Mathematically, it reduces to the following:

$$g_1 \equiv \quad 18\text{-}\dddot{Q}_{12} \geq 0 \tag{4.17}$$

II) Natural frequencies of the various subsystem affects the performance of the car. The bounce frequencies of front and rear suspension and that of pitching are most important. These frequencies can be calculated by using the Guest model [3] (Figure 4.2). Based on this model, the frequencies are given as follows:

Front suspension frequency in bounce

$$F_f = \frac{1}{2\pi}\sqrt{\frac{2K_{fs}(a-x+s)^2 + 2K_{rs}(s-b-x)^2}{M(\rho^2+s^2)}} \tag{4.18}$$

Rear suspension frequency in bounce

$$F_r = \frac{1}{2\pi}\sqrt{\frac{2K_{fs}(a-x-r)^2 + 2K_{rs}(r+b+x)^2}{M(\rho^2+r^2)}} \tag{4.19}$$

Frequency in pitching

$$F_p = \frac{1}{2\pi}\sqrt{\frac{2K_{fs}(a-x)^2 + 2K_{rs}(b+x)^2}{M\rho^2}} \tag{4.20}$$

where

x= distance between center of gravity and spring center

r, s = distance of front and rear conjugate points from spring center

a, b= distance of the spring center from front and rear

ρ= radius of gyration of sprung mass

These can be calculated by using following expressions (Refer Figure 4.2)

$$a = \frac{K_{rs}}{K_{rs} + k_{fs}} \times L$$

$$b = \frac{K_{fs}}{K_{rs} + k_{fs}} \times L$$

$$r = \frac{K^2 - x^2 - ab}{2x} - \sqrt{\frac{\{(K^2 - x^2 - ab)^2 + 4K^2x^2\}}{2x}}$$

$$s = \frac{K^2}{r}$$

Now using the expressions (4.18) to (4.20), constraints based on frequencies can be formulated.

i) Front suspension frequency in bounce should be less than the rear suspension frequency in bounce [10,17,19]. This is because, it will make pitch motion to die faster.

$$F_f < F_r$$

$$\text{or} \qquad g_2 \equiv F_r - F_f > 0 \qquad (4.21)$$

ii) The natural frequency of human body lies between 1.5 Hz to 2 Hz [11,17]. So any of the natural frequency of any subsystem in a car should not be greater then 1.5 Hz. This fact can formulated by putting the largest of these frequencies to be less then 1.5 Hz.

$$\max(F_f, F_r, F_p) \leq 1.5 \text{ Hz}$$

$$\text{or} \qquad g_3 \equiv 1.5 - \max(F_f, F_r, F_p) \geq 0 \qquad (4.22)$$

iii) To avoid a very small natural frequency any of these frequencies has been kept to be more than 0.8 Hz. This can be formulated as constraint by putting least frequency to be greater than 0.8 Hz.

$$\min(F_f, F_r, F_p) \geq 0.8$$

$$\text{or} \qquad g_4 \equiv \min(F_f, F_r, F_p) - 0.8 \geq 0 \qquad (4.24)$$

This completes the formulation of the constraints. Thus , there are four constraints to the problem.

4.3 Summary

The optimal design problem or any optimizations problem can be seen in three part, namely design variables, objective function and constraints. Every parameter associated with the design can not be taken as design variable. Few parameters which are most critical for the design are to be chosen as design variables. Formulation of the objective function requires the understanding of the problem so

as to choose an appropriate criterion as an objective function. The constraints but some sort of boundaries to the search space, as the acceptance of a feasible solution depends on them.

Objective functions for both two-dimensional and three-dimensional models are formulated for solving the governing differential equations in a reduced form. Two-dimensional problem has one constraint, restricting the jerk experienced. The objective is to minimize the transmissibility of the road irregularity to the sprung mass. Design variables are the damper coefficients and spring stiffens at front and rear suspensions.

Three-dimensional model has considered bouncing, pitching and rolling motion of the sprung mass. So, it can give more practically applicable results. The number of constraints for the three-dimensional problem are increased to four by putting constraints on the natural frequencies of the model. Three different objective functions are considered for these motions. The design variables are same as two-dimensional because the left and the right suspensions are usually the same. The NLP problem for the three-dimensional case can be written as follows:

minimize Equation (4.15) ,(4.16) or (4.17)

subjected to

$$18-\dddot{Q}_{12} \geq 0$$

$$1.5 - \max(F_f, F_r, F_p) \geq 0$$

$$F_r - F_f > 0$$

$$\min(F_f, F_r, F_p) - 0.8 \geq 0$$

$0 \leq K_{fsr}, \leq 3.0$

$0 \leq K_{rsr1} \leq 3.0$

$0 \leq \alpha_{fr1} \leq 3.0$

$0 \leq \alpha_{rr1} \leq 3.0$

In the next chapter, we solve the above two-dimensional and three-dimensional problems using a genetic algorithm.

Chapter 5

Simulation Results

In this chapter, results of the optimization carried out for the design of suspension system are presented. Optimization results for the two-dimensional model using traditional methods and using genetic algorithm are presented first. The proof-of-principle studies for the working of the code developed for the simulation and genetic algorithm are also presented. Thereafter, optimal design of the three-dimensional model is presented. Different motions of the sprung mass are plotted for the solutions found with different objective functions discussed in the previous chapter. The results are also compared with the design currently used at TELCO, Pune. Modeling of the a real road as a polyharmonic function is assumed and subsequent studies for

finding the optimal suspension design for maximum comfort as suggested in ISO 2631 are also discussed.

5.1 Two-dimensional Model

With the help of the two-dimensional model of the suspension system discussed in Chapter 2, the motion of the car over a bump (simulated with height 70 mm and breath 500 mm) is simulated and optimal suspension system design is achieved via solving the NLP problem discussed in Chapter 4. In this NLP problem, we have four design variables and one constraint. The objective is to achieve the minimum transmissibility of the road irregularities to the sprung mass. We solve the problem with some traditional optimization techniques first and thereafter with genetic algorithm.

5.1.1 Traditional Methods

A number of traditional methods are available for solving the optimal design problems. To find optimal design of the car suspension system, we used *'constr'* routine of the 'MATLAB OPTIMIZATION TOOLKIT' [13]. This routine finds the constrained minimum of a function of many variables, starting at an initial estimate. Mathematically, the problem should be in the following form:

$$\text{Minimize } f(x_1, \ldots x_n)$$

Subjected to

$$g_j(x_i) \leq 0 \qquad j=1 \text{ to } k$$

To put our problem in this form the constraint on the jerk is rewritten as follow :

$$\frac{18000}{\dddot{q}_2} - 1 \leq 0$$

This routine uses a Sequential Quadratic Programming(SQP) method. In this method, a Quadratic Programming(QP) subproblem is solved at each iteration. An estimate of the Hessian of the Lagrangian is updated at each iteration using BFGS formula [7,13]. A line search is performed using a merit function. The QP is solved using an active set strategy.

The governing differential equations of car dynamics are coded in MATLAB programming language and a MATLAB routine *'ode45'* is used to solve these equations to enable us to calculate the objective function and constraint values. The machine used for the purpose is a P.C. with PENTIUM processor running at 133MHz. Initially when the code was running with the default setting of accuracy in *'constr'*, the machine took seven hours but was not able to solve the problem. In many occasions in these seven hours, it prompted the massage

'Matrix is near singular or badly, scaled results may be inaccurate'.

Thereafter, when the option for the accuracy was set for a lower accuracy, the routine converged. In this way some results are found with different initial guesses. However, the results found by *'constr'* are very much dependent on the initial guess. Results with different initial points are shown in Table 5.1.

Table 5.1 Results Obtained With SQP method of MATLAB

Initial Solution						Solution Found					
K_{fs}	α_{f1}	K_{rs1}	α_{r1}	Trans.	Const.	K_{fs}	α_{f1}	K_{rs1}	α_{r1}	Trans.	Const.
1.0	1.0	1.0	1.0	0.96	-0.999	4.31	0.23	0.83	4.99	0.49	-0.995
1.5	3.0	1.0	0.75	0.82	-0.998	1.85	2.97	0.1	0.34	0.57	-0.998
1.0	1.0	1.5	0.9	0.89	-0.997	3.76	2.89	3.10	2.19	0.48	-0.996

Since the SQP method converges to different solution when started from different initial solution, it can be concluded that either the function is highly non-linear and multimodal in nature or the algorithm is unstable and not efficient and suitable for solving this problem. Whatever may be the case, the algorithm also takes a considerable amount of time in converging to the solution. Thus, we decided to use GA, an efficient optimization method, which could perform well in nonlinear, and multimodal problem and does not take too many function evaluations to converge to a near-optimal solution.

5.1.2 Genetic Algorithm (GA)

Difficulties encountered in solving the NLP problem for the two-dimensional model with traditional optimization techniques turned us towards the genetic algorithm. Before starting the optimization procedure with GA, we perform a couple of proof-of-principle results.

5.1.2.1 Proof-of-Principle Results

Contour plots are extensively used in optimization studies to clearly show the feasible region and the optimal solution. By plotting any solution on the contour plot, one can easily verify the optimality of the solution. We follow this simple strategy to verify the algorithm and the code developed for the simulation of the car's motion over the same bump, as used in the traditional case. It is easy to draw contour plots of the objective function on a two-dimensional plane. In our problem, there are four variables (K_{fs}, α_{f1}, K_{rs1}, and α_{r1}). Since we can only vary two variables at a time to take advantage of counter plotting , there are total six combinations possible. We investigate two of these combinations. In both cases, the following procedure is performed:

1. Only two of the four variables are varied and other two are kept fixed at certain values.

2. A contour plot of the objective function (transmissibility of the system in terms of vertical displacement) is first made by using an exhaustive search technique [7].
3. The feasible region corresponding to the constraint on jerk is determined by using the exhaustive search technique and plotted on the contour plot of the objective function.
4. From the feasible region on the contour plot the optimal solution is located by inspection.
5. The optimization code is used for two variables only to find the optimal solution and the corresponding optimal objective function value.
6. The optimal solution found using the optimization code is plotted on the counter plot to investigate the efficiency of the algorithm and working of the code.

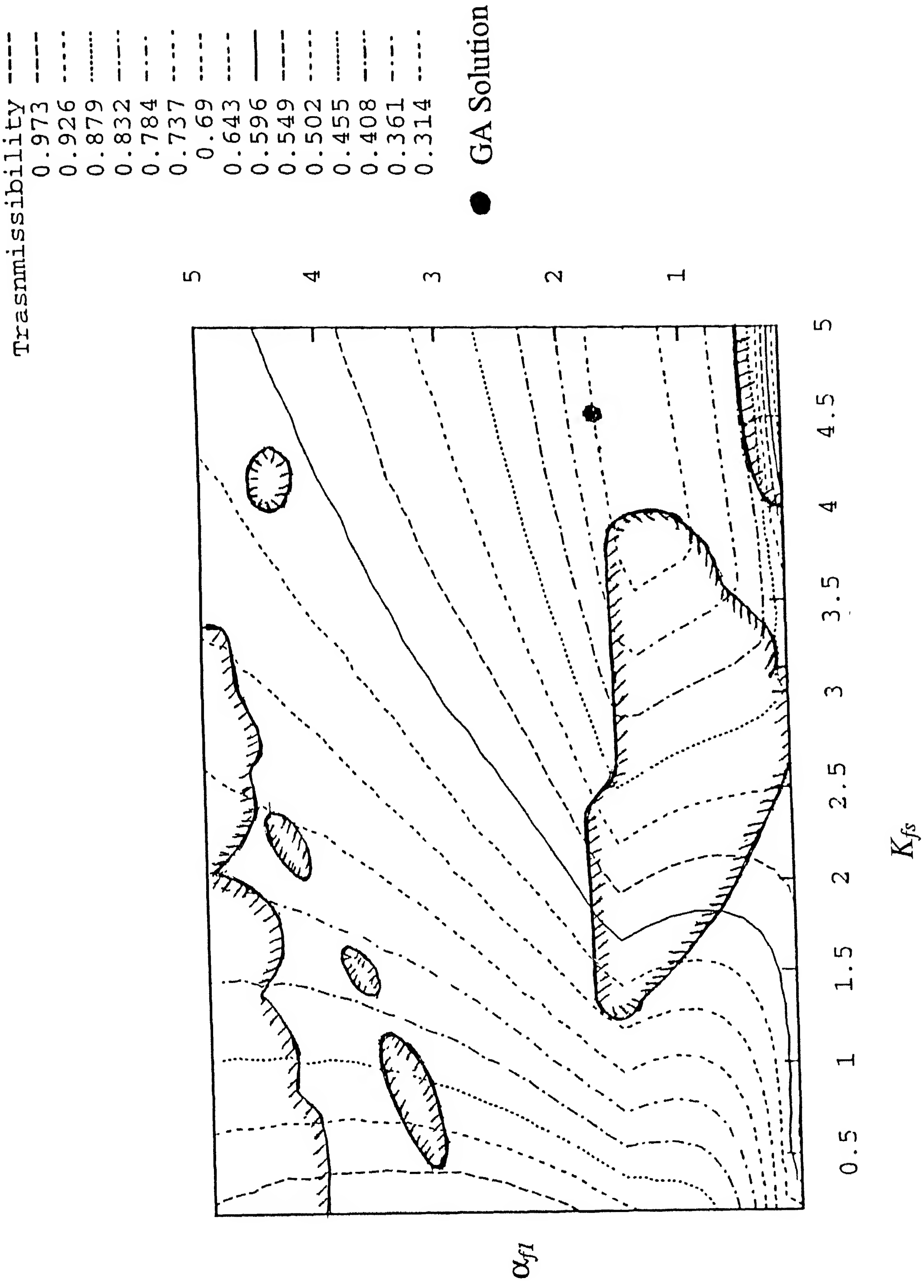
5.1.2.1.1 Varying Front Suspension Parameters

In the first case, front spring stiffness (K_{fs}) and front damper coefficient (α_{f1})are varied and the rear suspension's spring and damper are fixed at following values:

$$K_{rs1} = 2.86 \ kg/mm \qquad \alpha_{r1} = 1.01 \ kg\text{-}s/mm$$

Figure 5.1 shows the contour plot of transmissibility with respect to different front suspension parameters. The feasible region on the plot satisfies the constraint for the jerk. The Figure 5.1 shows that the solution space (feasible region) is disjointed. It has some infeasible region bounded by feasible region and vise versa. Multimodality of the feasible region comes from this disjointiveness and makes the problem difficult for traditional methods. The GA is used next to find the optimal solution of this two variable problem. The parameter used for the GA run are as follows:

Lower and upper bounds on variable one (K_{fs}) 0.1 and 5.0



Fiigure 5.1 Contours of Transmisibility and feasible region (by varying front parameters)

Lower and upper bounds on variable two (α_{f1})	0.1 and 5.0
Population size	: 30
Chromosomlength for variable one (K_{fs})	: 8
Chromosomlength for variable two (α_{f1})	: 8
Maximum number of generations	: 30
Crossover probability	: 0.9
Mutation probability	: 0.01
Tournament size for selection	: 2

The optimal solution found by the GA is as follows:

$$K_{fs} = 4.53 \ \ kg/mm, \qquad \alpha_{f1} = 1.72 \ \ kg\text{-}s/mm$$

The solution found by the GA is plotted in Figure 5.1. The solution seems to be close to the true optimal depicted by the feasible region shown in Figure 5.1. Thus, it can be argued that GA has found a near-optimal solution. We vary the other two variables, next and repeat the above experiment.

5.1.2.1.2 Varying Rear Suspension Parameters

The variables for the front suspension's spring and damper are fixed at following values:

$$K_{fs}= 4.53 \ kg/mm \qquad \alpha_{f1} = 1.72 \ \ kg\text{-}s/mm$$

The transmissibility value with respect to rear suspension parameters is shown as a contour plot in Figure 5.2. The feasible region on the plot satisfies the constraint for the jerk. The GA is used to find the optimal solution of this two-variable problem. The parameter used for the GA run are as follows:

Lower and upper bounds on variable three (K_{rs1})	0.1 and 5.0
Lower and upper bounds on variable four (α_{r1})	0.1 and 5.0
Population size	: 30
Chromosomlength for variable three (K_{rs1})	: 8
Chromosomlength for variable four (α_{r1})	: 8
Maximum number of generations	: 30
Crossover probability	: 0.9
Mutation probability	: 0.01
Tournament size for selection	: 2

The optimal solution found by the GA is as follows:

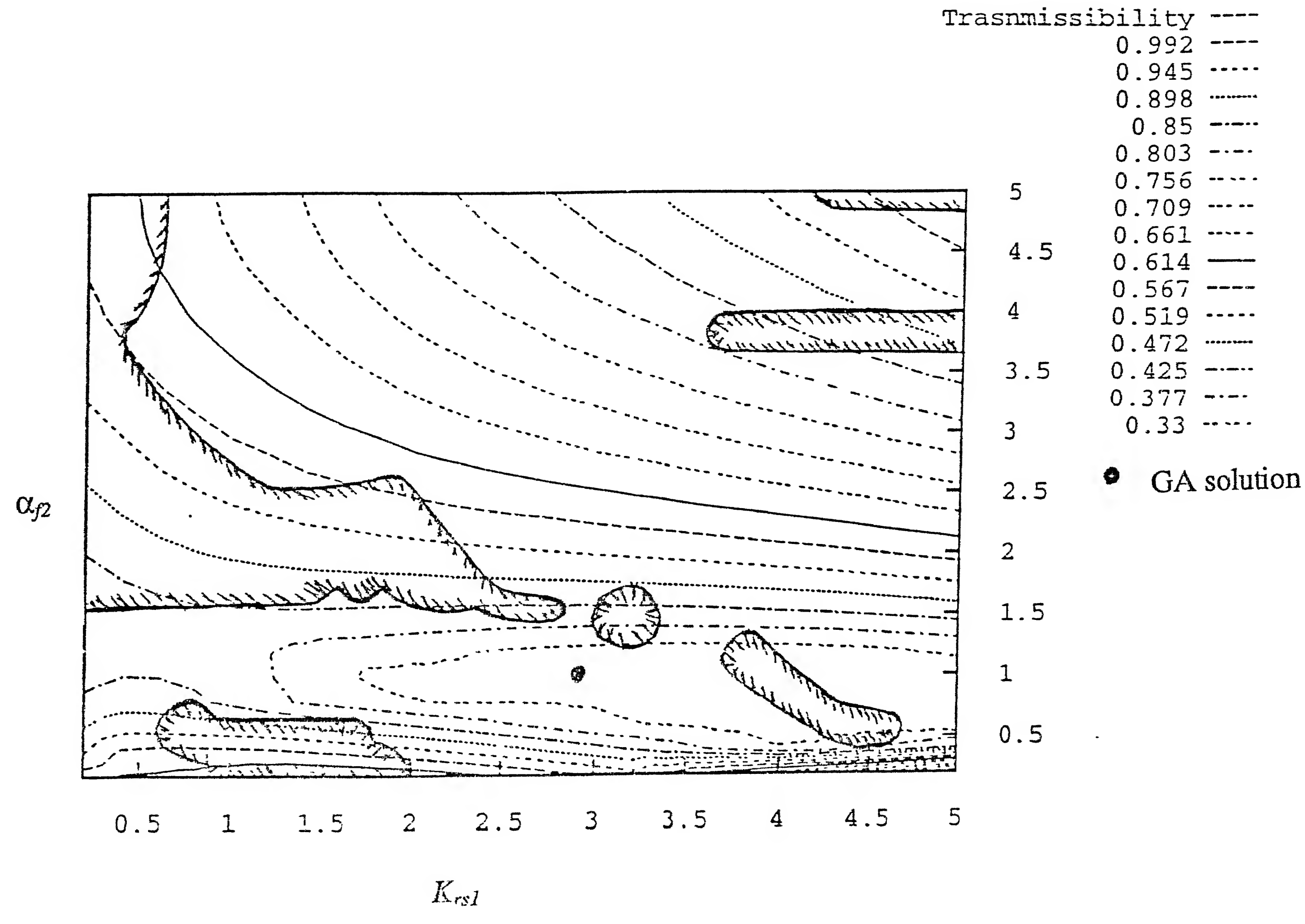
$$K_{rs1} = 2.85 \ kg/mm, \qquad \alpha_{r1} = 1.02 \ kg\text{-}s/mm$$

The optimal solution found by the GA and the optimal solution that can be visually inspected by from contour plot agree each other. Thus, it can be said that the GA has found a solution close to the optimal solution.

The above results give us the confidence to use GA as an optimization technique for the car suspension problem having four variables.

5.1.2.2 Varying Both Front and Rear Suspension Parameters

In this run now all the four variables are kept as decision variables. All other suspension parameters of the model are kept according to the data given by TELCO, Pune. These are as shown in Table 5.2. The car is assumed to ride over the same bump as in traditional case.



Fiigure 5.2 Contours of transmisibility and feasible region (varying rear parameters)

Table 5.2 Parameters for two-dimensional Model

Front Tyre Stiffness	15 kg/mm
Front Unsprung Mass	50 Kg
Rear Tyre Stiffness	17 kg/mm
Rear Unsprung Mass	115 Kg
Sprung Mass	730 Kg
Axle To Axle Distance	2825 mm
Distance of C.G. From Front	1325 mm
Moment of Inertia	2.8944e+04 kg-mm^2

The parameters used in this run are shown below. A population size of 30 and maximum of 30 generations are found to be adequate for this problem. The string length for each variable is kept as 10, so there are 2^{10} or 1024 values possible for each variable. This can give $(2^{10})^4$ or $1.1*10^{12}$ different solutions for the problem. The crossover and mutation probabilities are kept in accordance with suggestions in GA literature [12].

Population size : 30

Chromosomlength for variable one(K_{fs}) : 10

Chromosomlength for variable two(α_{f1}) : 10

Chromosomlength for variable three (K_{rs1}) : 10

Chromosomlength for variable four(α_{r1}) : 10

Maximum number of generations	: 30
Crossover probability	: 0.9
Mutation probability	: 0.01
Tournament size for selection	: 2

GA is run on a HP-9000 machine, using 'UNIX C' compiler. Five different runs were performed with different random seed numbers. Figure 5.3, shows the progress of the population-best solution with number of generations. The figure shows that the best transmissibility reduces continuously with generation number. The optimal solution found has a transmissibility is equal to 0.32 and the suspension parameters are as follows:

K_{fs}= 4.53 *kg/mm* α_{f1} = 1.72 *kg-s/mm* K_{rs1} = 2.86 *kg/mm* α_{r1} = 1.01 *kg-s/mm*

Time taken to find the solution in 90 seconds (CPU time) .The constraint is not violated in this case. Since, with four variables contour plots can not be achieved, we show the superiority of the solution by comparing its performance on the dynamic response with that used by TELCO, Pune:

K_{fs} = 1.56 *kg/mm* α_{f1} = 3.3 *kg-s/mm* K_{rs1} = 1.45 *kg/mm* α_{r1} = 1.00 *kg-s/mm*

With these parameters the car motion is simulated for the first 10 seconds after the hitting of the bump by the front tyre. In Figure 5.4, the vertical motion of the sprung mass (bouncing motion) is plotted and in Figure 5.5, the pitching motion (angular displacement) of the sprung mass is plotted.

From the Figure 5.4 and 5.5 it can be easily seen that the vertical displacement through out the 10 second simulation (and hence the transmissibility) is less with the parameter values resulted from optimization through GA.

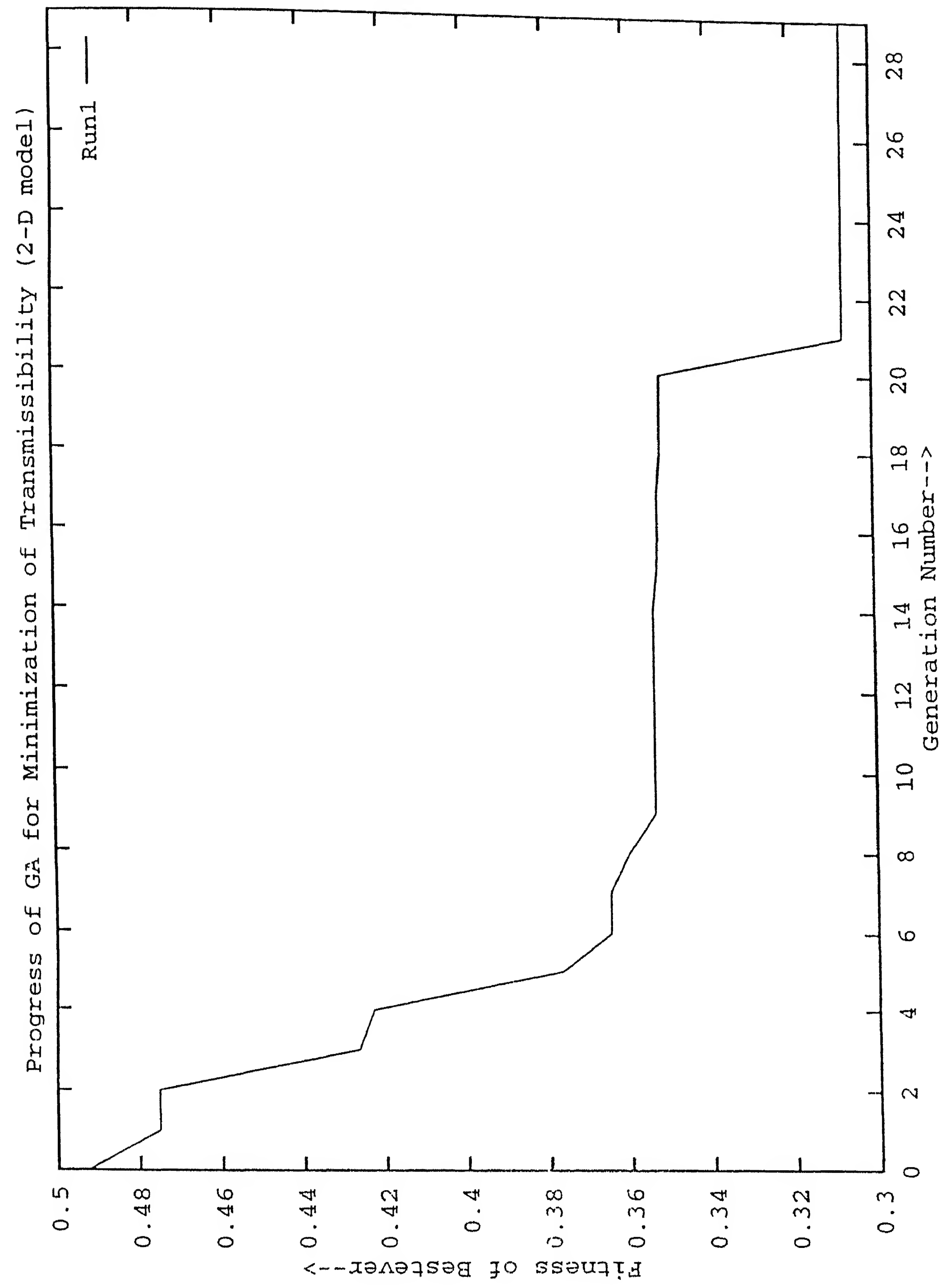


Figure 5.3 Progress of GA for Minimization of Transmissibility (2-D model)

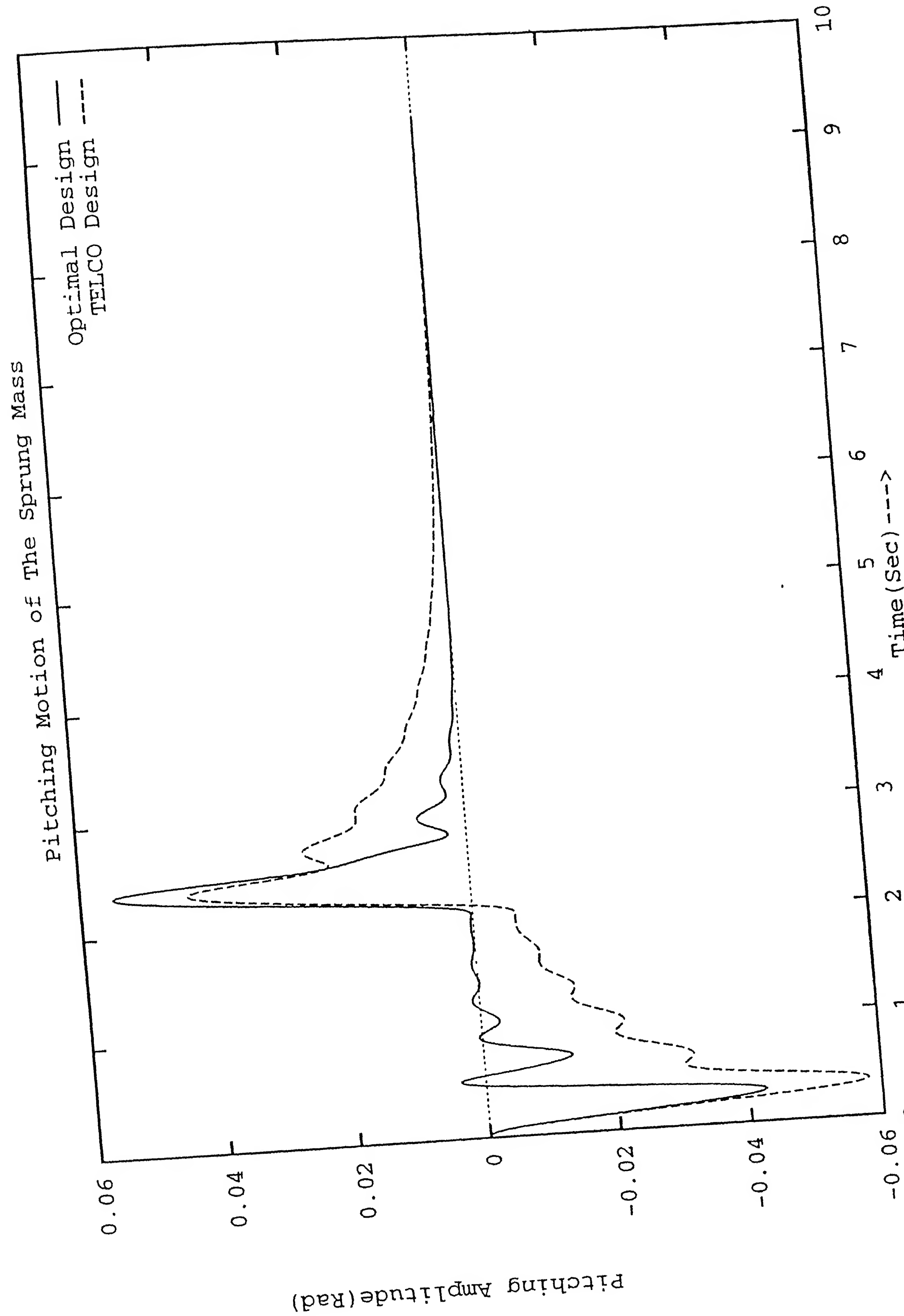


Figure 5.5 Motion of The Car Over a Bump (Pitching Motion)

The value of displacement transmissibility (Eq. 4.11)for the motion over a bump of 70 mm height and 500 mm spread is 0.82 with TELCO design. The same is reduced by about 62% to approximately 0.32, for the optimal suspension design. This reduction in transmissibility shows the effectiveness of the optimization procedure which is carried out with the help of GA.

With these encouraging results, achieved through two-dimensional model optimization. we carry out the optimization, and present simulation results for the three-dimensional model.

5.2 Three-dimensional model

To study the effect of various road conditions on suspension design, we have used three different excitation functions and three different objectives. In all the cases, the following GA parameters are used:

Number of runs	: 5
String length for variable K_{fs}	: 10
String length for variable α_{f1}	: 10
String length for variable K_{rs1}	: 10
String length for variable α_{r1}	: 10
Population size	: 30
Number of generations	: 30
Crossover probability	: 0.8
Mutation probability	: 0.01
Tournament size for selection	: 2

The best solution found in these runs is reported as the optimal solution. Simulation runs over different roads are presented next.

5.2.1 Over a Speed Breaker

In this case the car is assumed to move over a speed-breaker. The speedbreaker is modeled as a half sine wave with following specifications:

Height of the bump = 35 mm

spread of the bump = 100 mm

The GA is then used for minimization of the transmissibility defined on vertical displacement (Eq. 4.14), with constraints on jerk and frequencies (Eq. 4.17, 4.21, 4.22, 4.23). The progress of GA with generations is shown in Figure 5.6. The best solution found in the initial population had a non-dimensionalized transmissibility equal to 0.48. After 30 generations, the optimal solution found is as follows:

K_{fsr}= 1.54 *kg/mm* α_{fr1} = 2.63 *kg-s/mm* K_{rsr1} = 1.29 *kg/mm* α_{rr1} = 2.05 *kg-s/mm*

The corresponding value of transmissibility is 0.44. The bouncing motion of the sprung mass is plotted in Figure 5.7 and the pitching motion is plotted in Figure 5.8. The TELCO suspension design is an infeasible solution for this excitation function as the maximum value of the jerk is much more than the limit. The constraint violation by this solution is for jerk constraint, this give a jerk of 27000 mm/s^3. Since, the existing TELCO solution is not a feasible solution, it will not be justifiable to compare our solution with TELCO design.

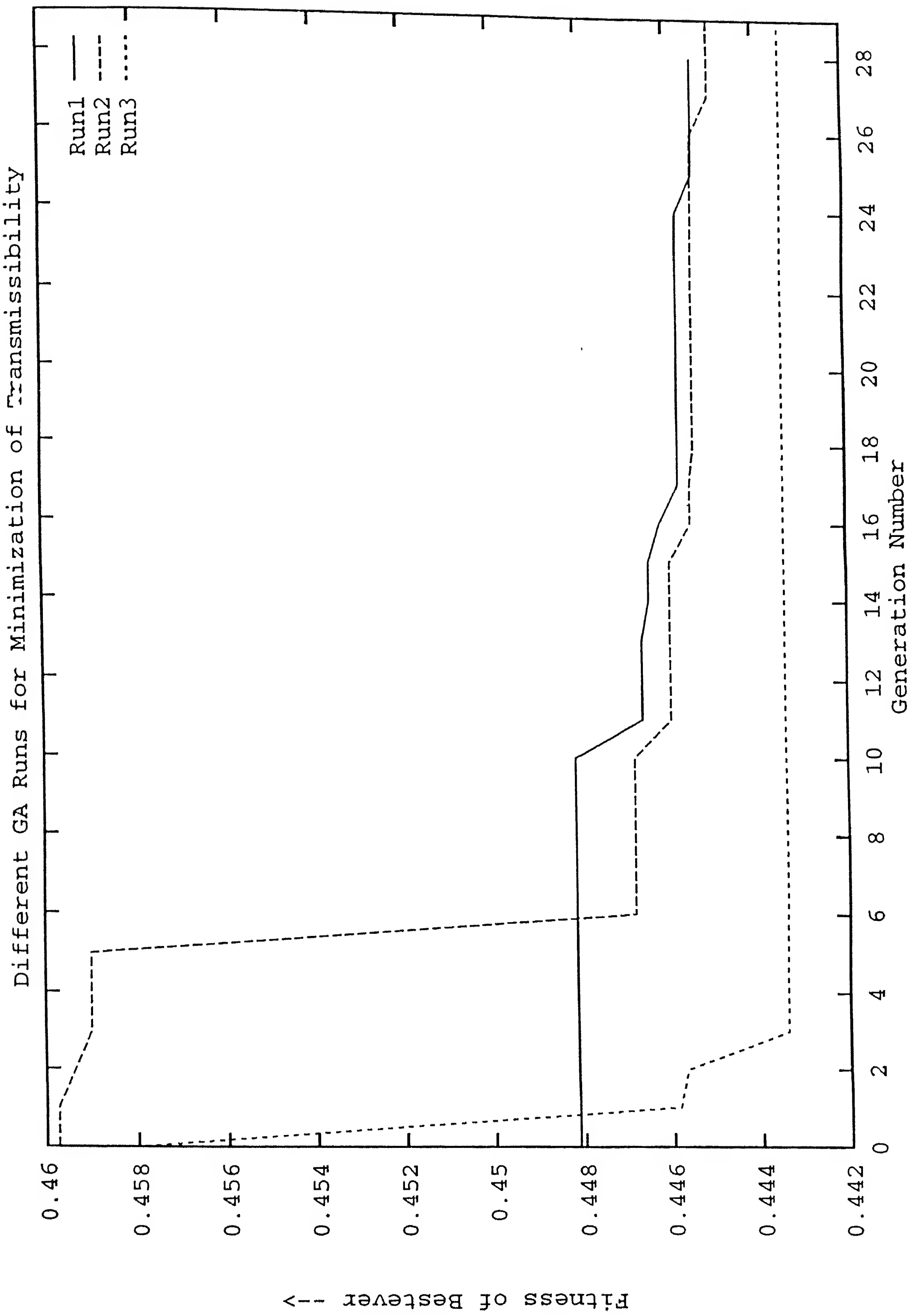


Figure 5.6 Progress of GA for Minimization of Transmissibility (3-D model)

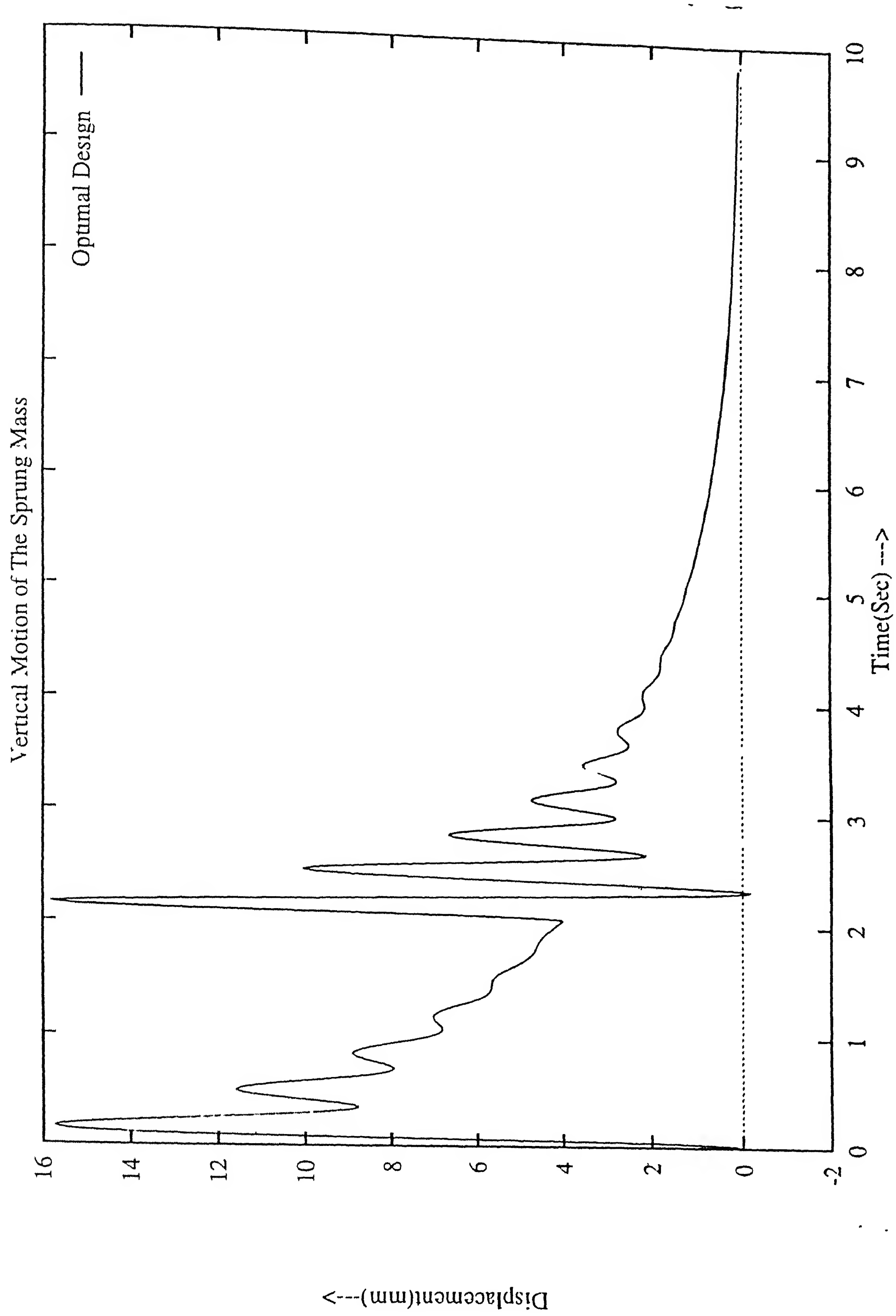


Figure 5.7 Motion of The Car Over a Speed-breaker (Bouncing Motion)

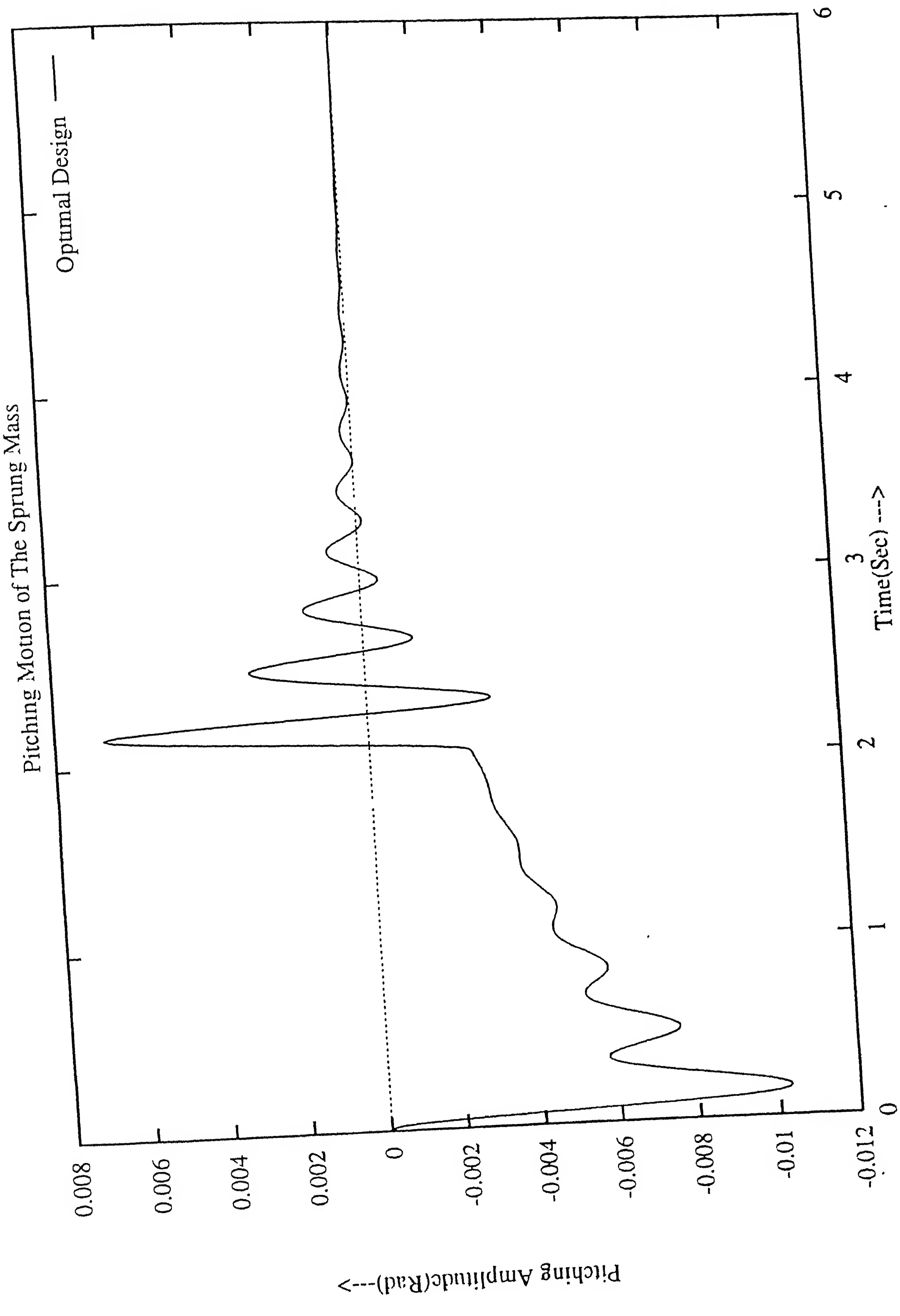


Figure 5.8 Motion of The Car Over a Speed-breaker (Pitching Motion)

5.2.2 Over a Sinusoidal Road

As discussed in section 4.2.3.2, a sinusoidal road is a road with the profile resembling a sine wave of large wavelength. The motion of the car moving with a velocity of 5 kmph on this type of the road is simulated. The road is defined using following specifications for the sine wave:

Wave length of the wave = 2850 mm

Maximum amplitude of the wave = 35 mm

The GA is then used for the minimizing the maximum pitching amplitude (Eq. 4.15), with Eq. 4.17, 4.21, 4.22, 4.23 as constraints on jerk and frequencies. The GA progressed towards the optimal solution is shown Figure 5.9. The optimal solution found after 30 generations is as follows:

K_{fsr}= 0.84 *kg/mm* α_{fr1} = 1.77 *kg-s/mm* K_{rsr1} = 0.83 *kg/mm* α_{rr1} = 1.21 *kg-s/mm*

The Figure 5.10 shows that GA has reduced the pitching transmissibility over a few generations. All three motions of the sprung mass, namely bouncing, pitching and rolling with suspension system parameters for this design are shown in Figures 5.11, 5.12 and 5.13, respectively (Presented after discussion of the subsection 5.2.4).

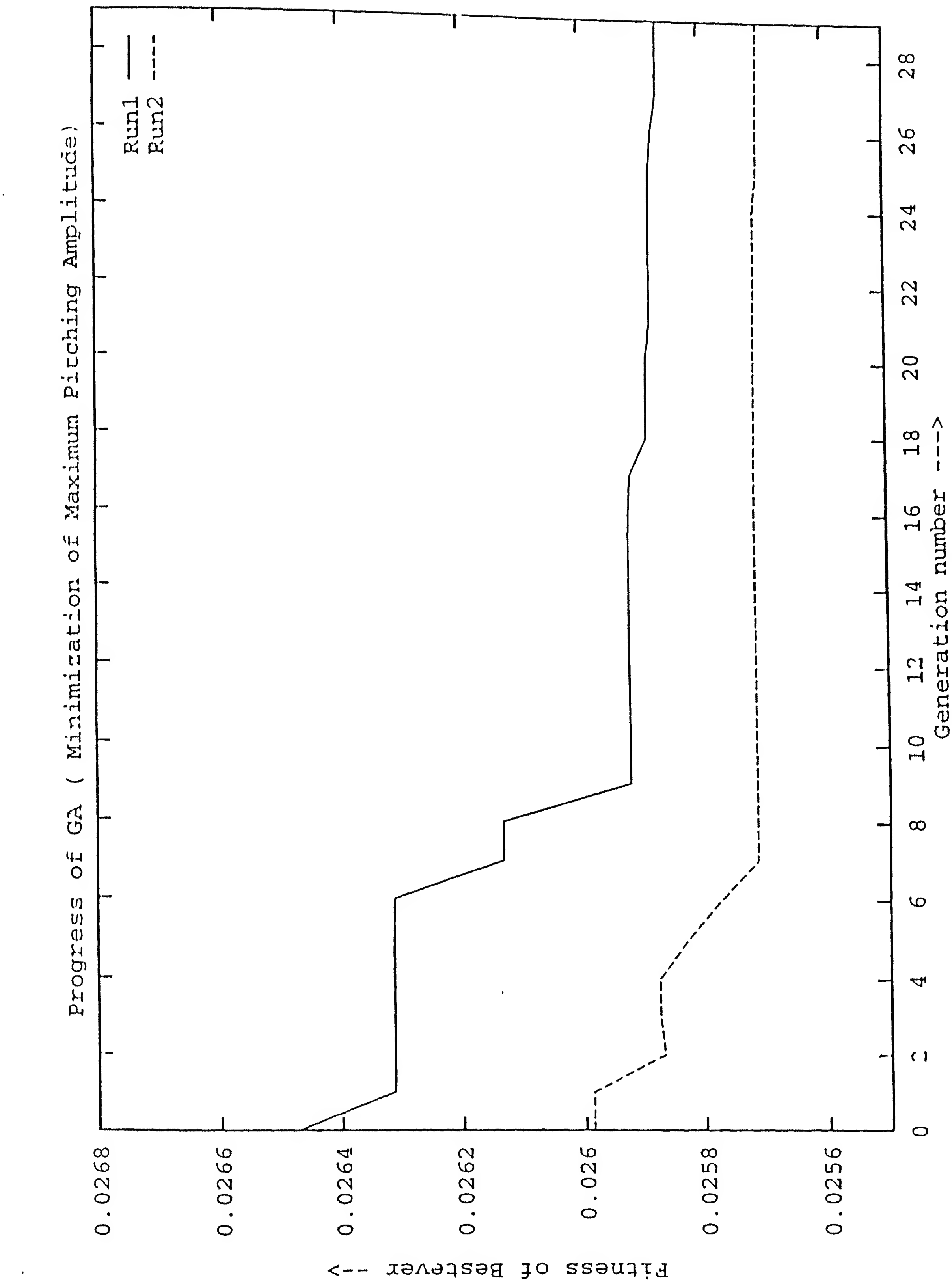


Figure 5.9 Progress of GA for Minimization of Maximum Pitching Amplitude

5.2.3 Over a Twisting Road

Rolling motion of a car will be most severe when the car is moving over a twisting road. On a twisting road, right and left wheels moves out of phase on two sinusoidal road profile. Car's speed is again taken as 5 kmph and motion on such roads of the following sine wave specifications is simulated:

Wavelength = 2850 mm

Maximum Amplitude = 35 mm

With the objective of minimization of the rolling amplitude(Eq. 4.16), GA runs are performed. Constraints on jerk and frequencies are taken as in the previous cases. The best solution found (which gives the maximum rolling amplitude of 0.032132 radians) is as follows:

K_{fsr}= 0.64 *kg/mm* α_{fr1} = 1.02 *kg-s/mm* K_{rsr1} = 0.43 *kg/mm* α_{rr1} = 1.33 *kg-s/mm*

Figure 5.10 shows the progress of GA with the generation while minimizing the maximum amplitude of rolling. Bouncing, pitching and rolling motions of the sprung mass with above values as the suspension parameters are shown in Figures 5.11, 5.12 and 5.13 respectively. (Presented after discussion on subsection 5.2 4)

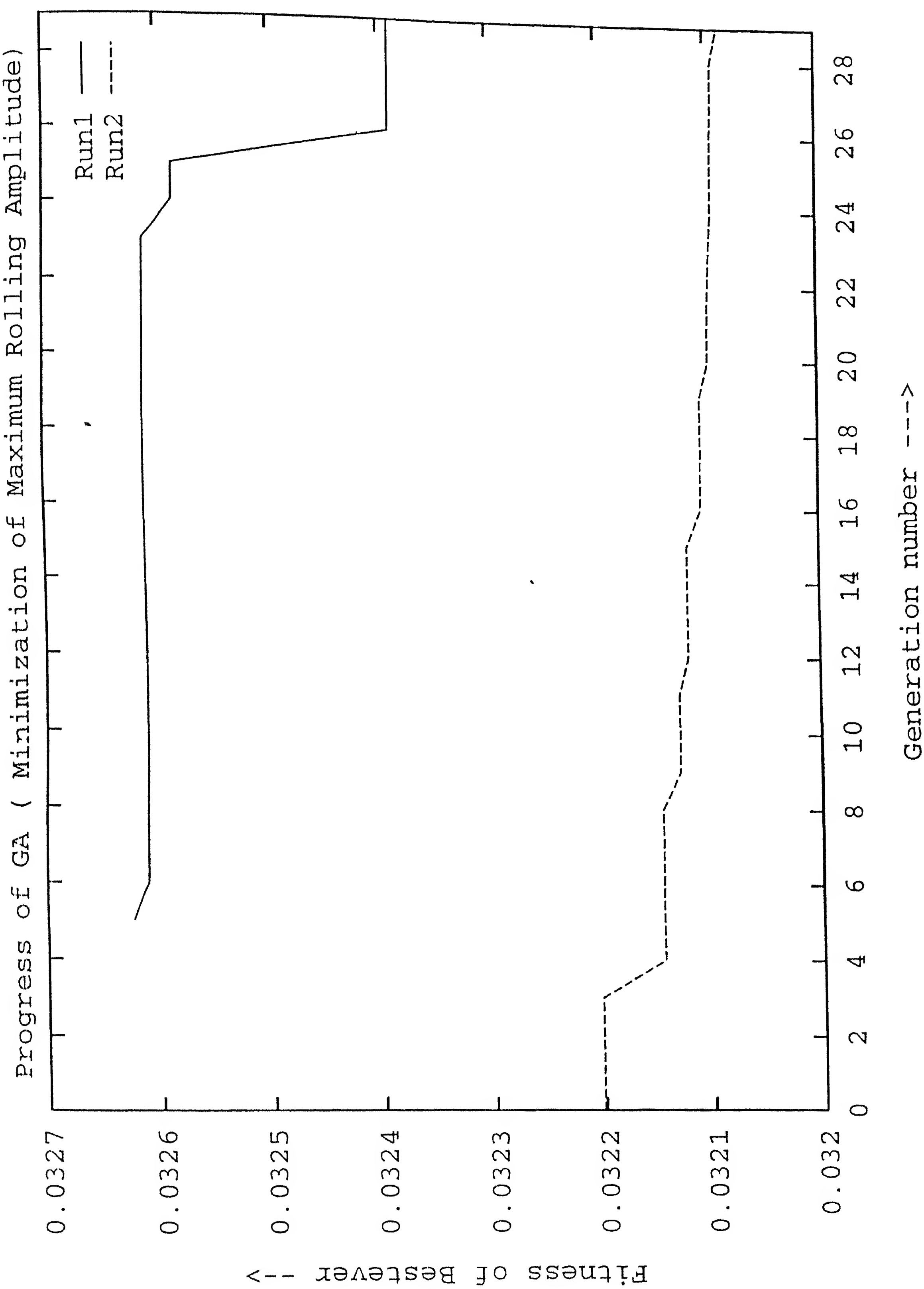


Figure 5.10 Progress of GA for Minimization of Maximimum Rolling Amplitude

5.2.4 Over A Combined Road

Above road profiles(bump, sinusoidal and twisting) are simple and makes the optimization procedure easier to converge. Therefore, they are not totally practical . A road can be better approximated by a combination of these three basic road profiles. Thus, we find the optimal car suspension for all the three road profiles simultaneously. The car's motion is simulated over each of the them and corresponding objective function value is calculated. These objective functions values are then combined to get an overall objective for the GA. GA runs are performed with the following parameters:

Maximum number of runs	: 5
String length for variable K_{fs}	: 10
String length for variable α_{f1}	: 10
String length for variable K_{rs1}	: 10
String length for variable α_{r1}	: 10
Population size	: 30
Maximum number of generations	: 40
Crossover probability	: 0.8
Mutation probability	: 0.01
Tournament size for selection	: 2

The best solution found in these runs gives the following values of the suspension parameters:

K_{fsr}= 1.43 *kg/mm* α_{fr1} = 2.46 *kg-s/mm* K_{rsr1} = 1.55 *kg/mm* α_{rr1} = 1.91 *kg-s/mm*

For this solution, the different objective functions have following value:

Transmissibility	= 0.48
Maximum pitching amplitude	= 0.0253
Maximum rolling amplitude	= 0.03231

Figure 5 11 shows the bouncing motion of the sprung mass for the four solution found above- optimized for the bouncing only, optimized for pitching only, optimized for rolling only, and optimized for all of them together.. The lowest value of transmissibility, as expected is for minimization of bouncing case. The combined solution gives slightly higher value for the transmissibility. The other two solutions have even higher vertical transmissibilities.

Figure 5.12 depict the pitching motion of sprung mass for all the four solutions. Maximum pitching amplitude remains more or less the same. It is slightly higher for the solution obtained for the combined run.

Figure 5.13 gives the rolling motion of the, sprung mass for all of the solutions found above. Maximum rolling amplitude is more or less same in all cases but it is slightly higher for the solution obtained for the combined run.

The above study for various objectives and road profiles reveal the fact that if optimization is done only for minimization of the transmissibility of vertical displacement, other two objectives and road profiles need not to be optimized. The optimization of vertical displacement transmissibility inherently optimizes other cases. Thus, designers of car suspension may only optimize for vertical displacement.

In next section we present, more practical approach to the suspension design.

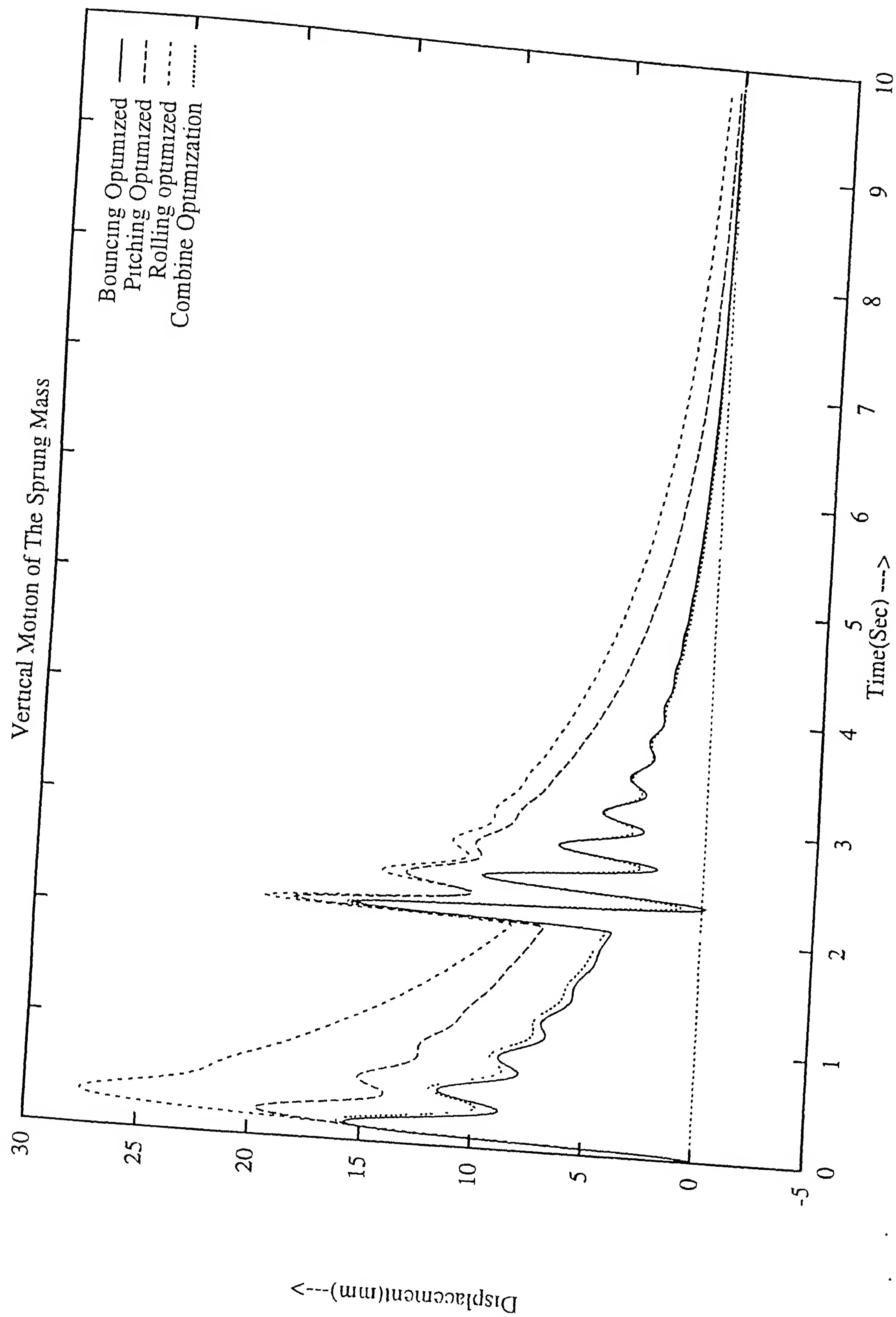


Figure 5.11 Motion of The Car over a Speed-breaker

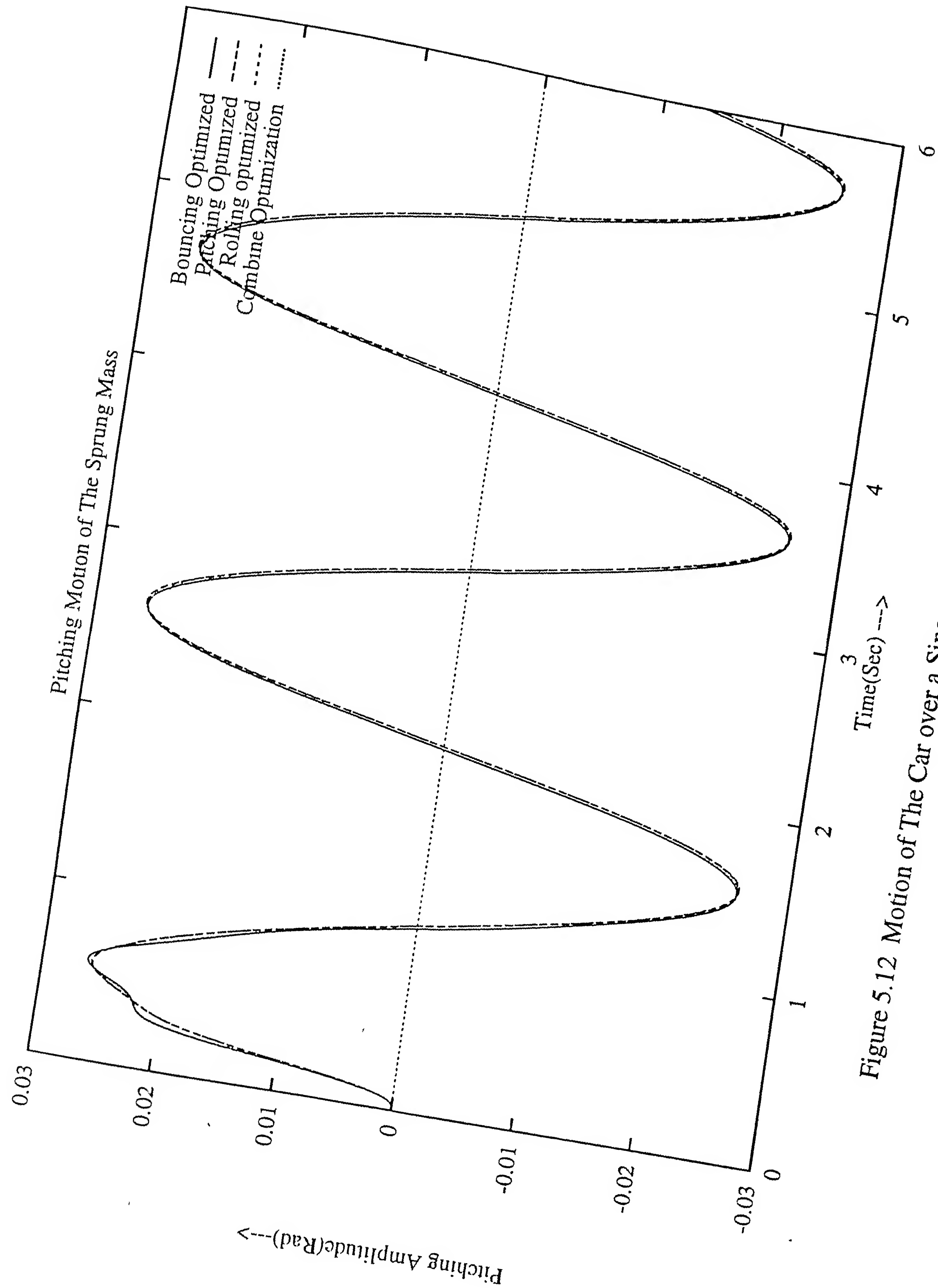


Figure 5.12 Motion of The Car over a Sine wave Road

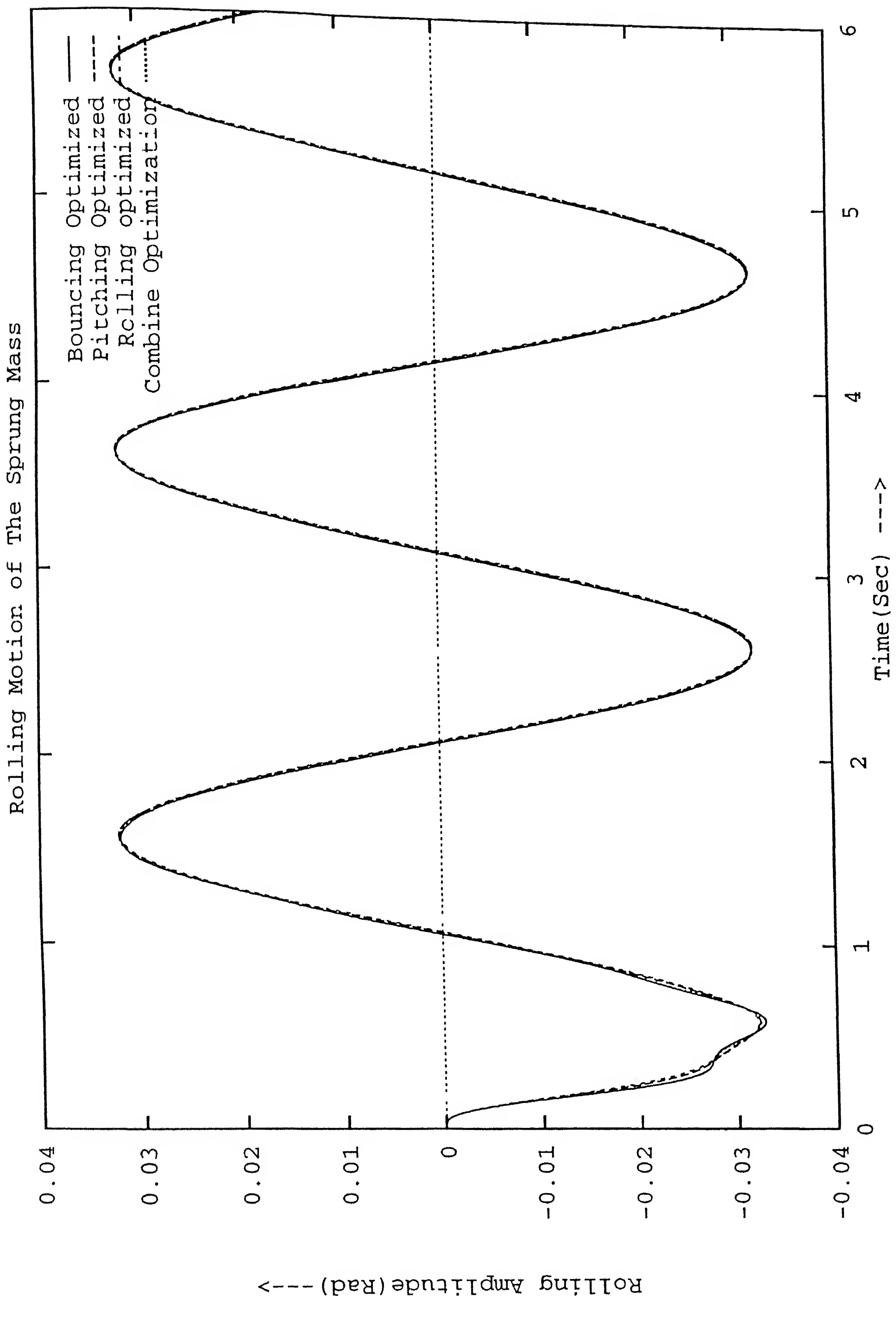


Figure 5.13 Motion of The Car over a Twisting Road

5.2.5 Realistic Road and Comfort Ride

In practice, the roads are neither of the types discussed earlier. The road undulations vary randomly in sizes and shapes. To simulate such a road, researchers have assumed a combination of sinusoidal road excitation having different frequency, phase and amplitude [9,16,20]. For this purpose, a polyharmonic excitation function is chosen which can be expressed as follows:

$$f(t) = \sum_{\lambda=\lambda_1}^{\lambda_n} A(\lambda)\sin[2\pi\frac{v}{\lambda}t - \phi(\lambda)], \tag{5.1}$$

where

$A(\lambda)$= Amplitude spectrum of road roughness

λ = wavelength

t = time

$\phi(\lambda)$= phase angle dependent on λ

The wavelength and amplitude has a linear relationship [9], which can be expressed as follows:

$$A(\lambda)=A_0 + B_0\,\lambda$$

where A_0 and B_0 are the coefficient depending on the type of the road. Wavelengths in a ordinary road generally vary between 100 mm to 5000 mm. Hence, value of λ_1 in equation(5.1) is taken as 100 and λ_n is taken as 5000.

The phase angle is defined by the following expression:

$$\phi(\lambda) = 2\pi(RND - 0.5),$$

where *RND* stands for a set of random number in the interval [0,1]. Now, the excitation function for the front wheels can be written as follows:

$$f_1(t)=f_2(t)=f(t)$$

and for the rear wheels

$$f_3(t)=f_4(t)=f(t-L/v)$$

A profile of such a road created with above function is shown in Figure (5 .14).

Passengers and driver of a car moving over this road will be subjected to vibrations varying in frequency and amplitude. The extent of vibration imparted to the passenger determines the comfort of the occupants of the car. Various methods of rating the severity of the exposure have been developed for specific applications. But, none of this methods are universally accepted [15]. In view of the complex factors determining the human response for vibration, international standards has been prepared to facilitate the evolution and comparison of the data and to give provisional guidance for suspension design. The standard ISO 2631 defines and gives numerical values for the limits of exposure of vibration transmitted to the human body in frequency range of 1 to 80 Hz. This limits are given for the use according to the three generally recognizable criteria of preserving comfort, working efficiency, and safety or health. The limits set according to these criteria are named respectively in ISO 2631/1 the 'reduced comfort boundary', 'fatigue-decreased proficiency boundary' and the 'exposure limit'. In the design of passenger accommodations, the 'reduced comfort boundary is recommended as per ISO 2631/1.

These limits are specified in terms of vibration frequency, acceleration magnitude, exposure time and direction of the vibration. Our simulation of the car motion gives only the displacement and velocity of the responses with time. Hence, the acceleration is calculated by central difference

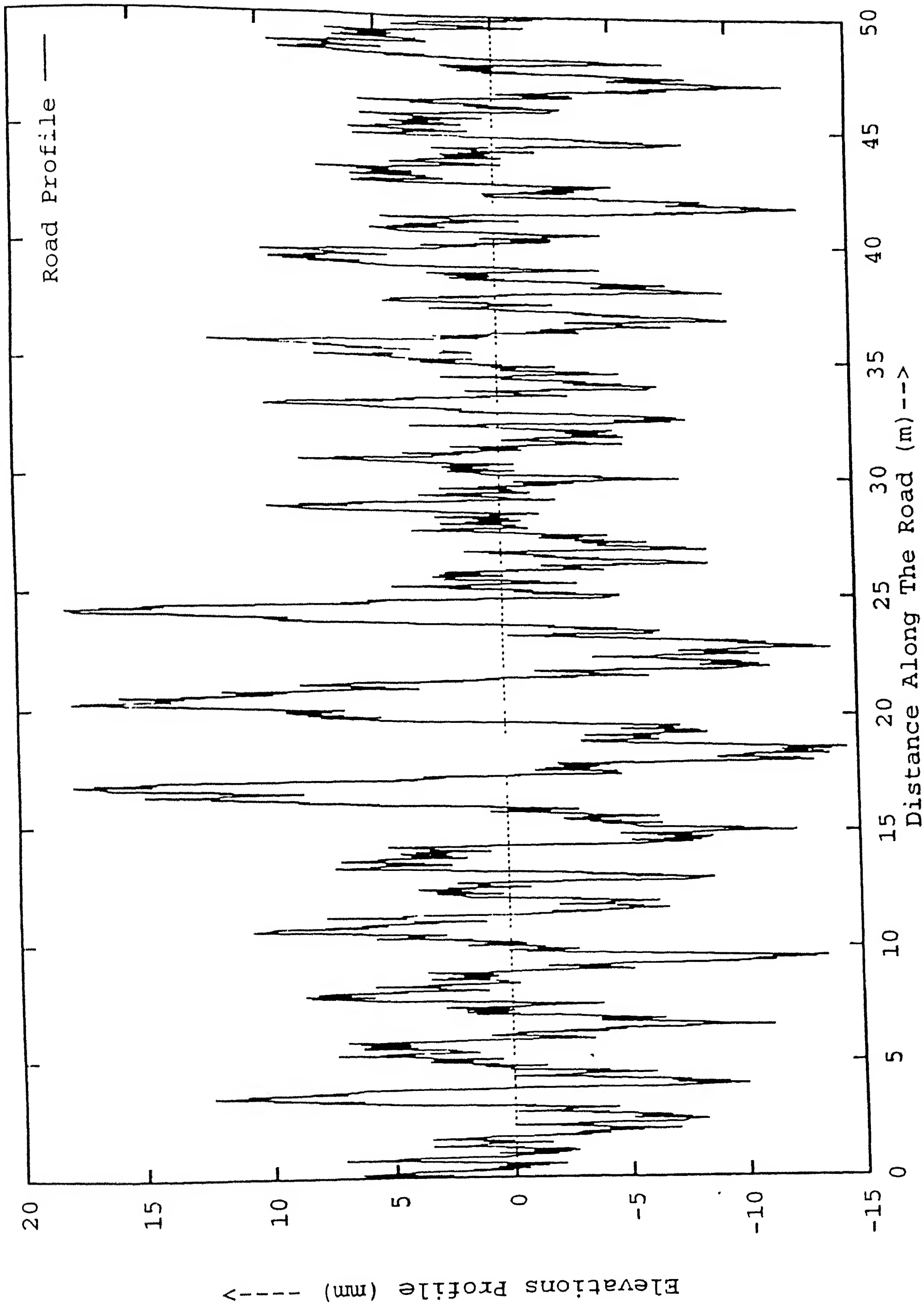


Figure 5.14 Elevation Profile of a Realistic Road (generated by Equation 5.1)

method and is Fourier transformed to get the frequency dependency of the acceleration. Now, the acceleration can be plotted with frequency in accordance with ISO 2631. One such plot for TELCO's design is shown in Figure 5.15. The plot shows that there are some distinct frequency levels at which the amplitude of acceleration is severe. Passenger comfort level increases with the reduction in these acceleration amplitudes at different frequencies. We exploit this fact to formulate the objective function for the maximization of the comfort. The procedure for calculating the objective function for a set of suspension parameters is as follows:

1. For a suspension system, the three-dimensional dynamic response of the sprung mass is found for motion over the random road profile.
2. Acceleration of the sprung mass is calculated using velocity of sprung mass history by central difference method.
3. Fourier transformation of the acceleration data is done to find frequency-acceleration data.
4. Peaks of the frequency-acceleration plot are recorded.
5. Sum of the peaks of acceleration for different frequencies is taken as objective function value.

With this objective function and constraints as in previous cases (jerk and frequencies), the GA is used to get a optimal design (for minimum objective function value). The GA parameters similar to the previous cases are used to get the optimal solution. The optimal solution found after 30 generations is as follows :

K_{fsr}= 1.45 *kg/mm* α_{fr1} = 0.14 *kg-s/mm* K_{rsr1} = 1.28 *kg/mm* α_{rr1} = 0.11 *kg-s/mm*

It is surprising that the damping coefficients values in this solution are very close to their lower bounds. This perhaps has happened because we don't have any velocity related constraint. Dampers are sensitive to velocity and since none of the constraints directly relate to velocity, the

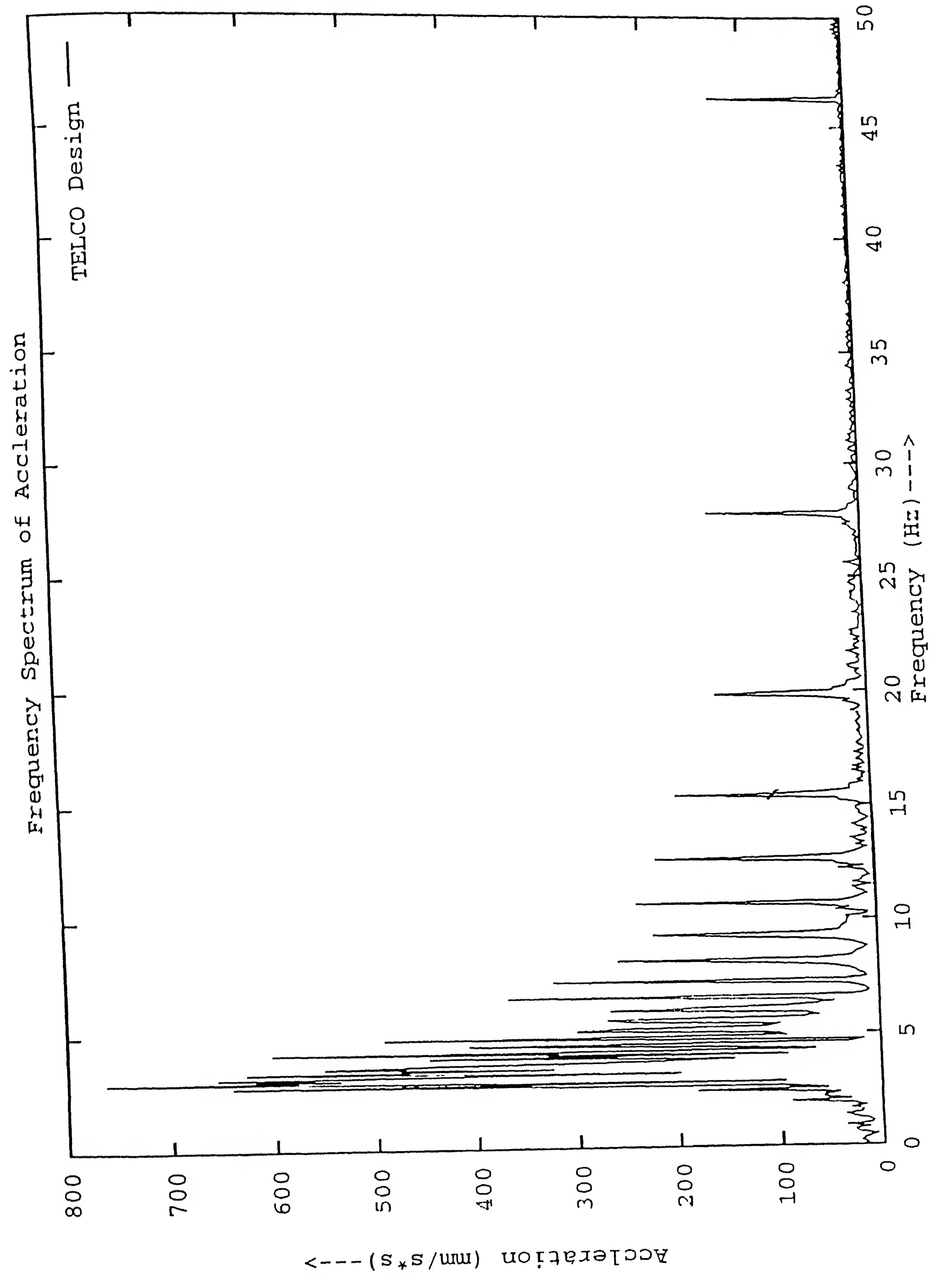


Figure 5.15 Vertical Accleration of Sprung Mass- Frequency Spectrum

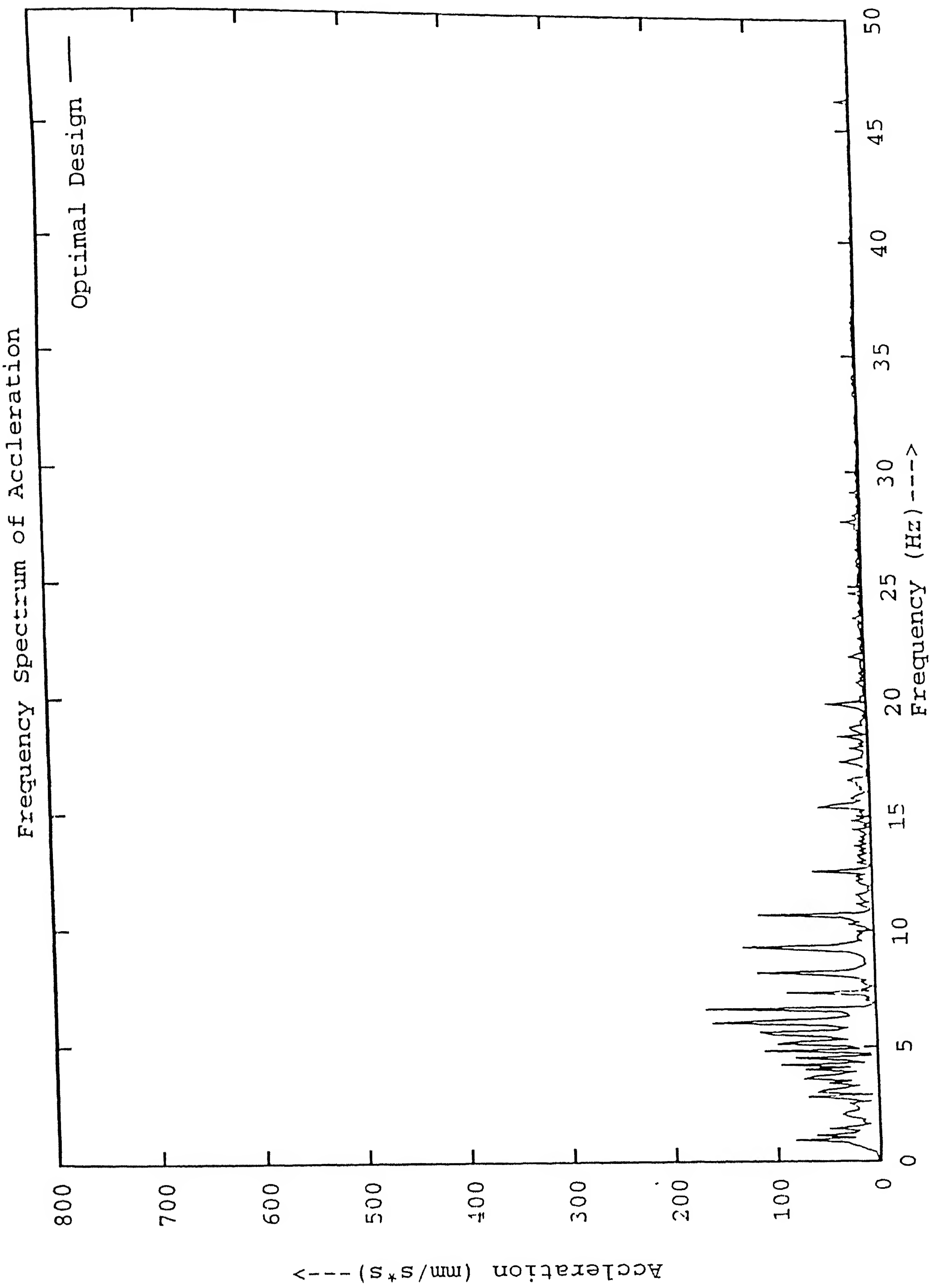


Figure 5.16 Vertical Accleration of Sprung Mass- Frequency Spectrum

optimization does not find the need of using a stiff damper. The frequency verses acceleration plot for this solution is shown in Figure 5.16. Which shows that the optimized suspension has an order of magnitude less acceleration than the TELCO design on the random road. Calculating comfort boundary from ISO 2631 chart, it is found that the car is comfortable for driving of 2.5 hours, in compare to only one hour in the case of existing TELCO design. This is a considerable improvement in the design of the car suspension. Table 5.3 shows an overall comparison of optimal suspensions obtained for different objectives and different road conditions.

5.3 Summary

Finding of all the optimization runs with different road excitations and different objective functions can be summarized in the following:

1. Traditional optimization methods take about seven hours on PC with a PENTIUM processor and still do not found a stable solution.
2. Optimal design of suspension found using GA, for 2-dimensional model gives 62% better performance than present TELCO design, in terms of vertical displacement transmissibility. Constraints on frequencies have some violation for this solution (table 5.3), but actually there were no such constraints on the design in 2-dimensional problem formulation.
3. It was found that TELCO design does not satisfy the constraint on the jerk, in speedbreaker case with 3-dimensional model. Optimal solution found using GA is feasible and takes only 90 seconds (CPU time) on a HP-9000 machine.

4. On a sinusoidal road, slightly better performance is achieved using GA, but the solution is closer to the optimal solution compared to the TELCO design, since the constraint values are close to zero.
5. On a twisting road, considerable improvement in the rolling performance is achieved using GA. The third constraint on the frequencies has a value of -0.002, because the natural frequency of the front suspension in bouncing is 0.79799 Hz, which is very close to 0.8 Hz, and as the constraints on the lower and upper bound of the frequencies are relaxible, this very small constraint violation does not makes the solution infeasible.
6. Optimal solution for the combined run shows a small violation of the constraint on upper bound on the frequencies because the natural frequency of the rear suspension in bouncing is 1.515 Hz for the solution found. This values is very close to 1.5 Hz. Again, as the constraints on the lower and upper bound of the frequencies are relaxible, this small constraint violation does not makes the solution infeasible.
7. On a random road, simulated by a polyharmonic function, the TELCO design fails to satisfy the jerk constraint. The maximum value of the jerk for TELCO design is approximately 45000 mm/s^3. The solution found using GA satisfies all the constraints and hence is a feasible solution.

This thesis considered many objectives and road conditions and found optimal suspension. Since, the optimal solutions vary for different objectives, the designer may choose the one that is most suitable for application. Most importantly, this thesis has demonstrated that any such objective and road condition can be considered in the optimal formulation of the car suspension designed the optimal solution can be found out within a reasonable amount of time.

Table 5.3 Summary of Results

	2-D Model		3-D Model									
	Speed Breaker		Speed Breaker		Sinusodial		Twisting		Combined	Random	
	Telco	Optimal	Telco*	Optimal	Telco	Optimal	Telco	Optimal	Optimal	Telco*	Optimal
K_{fs}	1.56	4.53	1.56	1.54	1.56	0.84	1.56	0.64	1.43	1.56	1.45
α_{f1}	3.30	1.72	3.30	2.63	3.30	1.77	3.30	1.02	2.46	3.30	0.14
K_{rs1}	1.45	2.86	1.45	1.29	1.45	0.83	1.45	0.43	1.55	1.45	1.28
α_{r1}	1.00	1.01	1.00	2.05	1.00	1.21	1.00	1.33	1.91	1.00	0.11
Constraints											
Jerk	0.519	0.717	-0.58	0.0116	0.548	0.0965	0.999	0.999	0.0711	-1.538*	0.2686
Fre1	0.062	-0.201	0.062	0.0121	0.062	0.0783	0.062	0.108	0.1702	0.062	0.0130
Fre2	0.034	-1.52	0.034	0.1056	0.034	0.3913	0.034	0.600	-0.0150	0.034	0.1339
Fre3	0.603	1.52	0.603	0.5821	0.603	0.2297	0.603	-0.002	0.5401	0.603	0.5529
Objective Function											
Trans. of Vertical Displ	0.82	0.318	0.402*	0.445	-----	------	------	-------	0.489	0.402	0.827
Pitching	0.058	0.057	-------	-------	0.026	0.0258	------	------	0.048	0.026	0.035
Rolling	-----	-----	------	-------	-----	------	0.044	0.013	0.039	0.044	0.066

* Infeasible Design

Chapter 6

Conclusions And Extensions

Based on this study we make some conclusions and then present some extensions.

6.1 Conclusions

Suspension systems of any vehicle is primarily used to isolate as far as possible the passenger compartment from the road undulations. Application of vibration theory helps us in understanding the vehicle dynamic response of a car to the road excitations. Transmissibility of the displacement has been considered as a criterion for the objective measurement of the vibrations. Two-dimensional as well as three-dimensional dynamic model of the car has been developed with front and rear both the suspensions as of independent type.

In this thesis the work has been aimed towards finding the optimal suspension design. It has been observed that the optimal suspension design problem is too complex and multimodal to solve

using a traditional optimization technique borrowed for MATLAB OPTIMIZATION TOOLBOX. The time taken by the machine was also considerably large. Genetic algorithm is then used to find the optimal solution. A number of different objectives have been used and a number of constraints are also checked as an acceptance criteria for the optimal solution. One constraint limits the value of the vertical jerk experienced by the passenger and three other constraints are related to the natural frequencies of the subsystems of the suspension system. Optimization problem formed in this way has four design variables corresponding to the front and rear suspension springs and dampers.

It was observed that for minimum transmissibility, the springs of the suspension system tend to be more stiff. This increases the natural frequencies of the front and rear suspension. But as in two-dimensional, case the jerk was the only constraint, optimal solution did not take care of natural frequencies of the system.

The three-dimensional model has been used next to find the optimal suspension system. Three different objectives, namely minimization of transmissibility, minimization of maximum pitching amplitude and minimization of maximum rolling amplitude have been used separately and the optimal solution is found for each of them. Thereafter, a combined objective function with a weighted average of above three objectives has been used to get the optimal design. The results suggest that minimization of transmissibility alone may be sufficient, because optimization of rolling or pitching amplitude results in more or less the same solutions. This happens due to the constraints on the frequencies. For rolling amplitude to be minimum, the spring stiffnesses will reduce to very low value, but lower bound on the frequencies keeps the stiffnesses above a certain limit and in turn keeps the maximum rolling amplitude on a particular level. On the other hand,

constraint of rear suspension frequency to be more then front suspension frequency takes care of pitching amplitude.

In order to model a realistic road, a random road, modeled as a polyharmonic function has been used to excite the car motion. In an objective of maximizing ride comfort defined as per ISO 2631 standard, GA has been able to find a much better solution compared to the TELCO design. In a nutshell, the following conclusions of the study can be made:

1. Optimal car suspension design problem (2-D or 3-D) give rise to a feasible search space which is highly complex, multimodal, and disjointed.
2. Popular traditional methods are not suitable to solve such problem. Although in some cases parameters can be adjusted to make the algorithm converge, the converged solution largely depends on the chosen initial solution. Moreover, the time taken to solve such problem is enormous.
3. Genetic Algorithms (GAs) can solve such problem quite efficiently, because of their population approach and use of only function values. Moreover, the time taken to such problem is also reasonable.
4. GAs have been used to solve the three-dimensional model of optimal suspension design problem for different road excitations and for three different objectives — vertical displacement, pitching, and rolling of sprung mass. GAs can repeatedly find the optimal solution. It is observed that the optimal solution for vertical displacement minimization for a car's ride over a bump is also nearly optimal for other objectives and for other road models.

5. A polyharmonic road model has been built to simulate a realistic road. GAs have been used to find optimal car suspension for maximum ride comfort as per ISO 2631. The optimal solution found is better than that used by TELCO.

Over all, it can be concluded that GAs are suitable to solve complex problem of optimal car suspension design. The success of GAs in this problem suggests their immediate application in other problems of engineering. Specifically, the following extensions can be immediately looked at.

6.2 Extensions

Findings and investigations during the course of this thesis work open up a number of avenues for immediate studies in the optimal suspension design. We present some of them in the following:

1. A more accurate and realistic model of the car for the suspension related studies can be formulated by assuming rigid body properties of the sprung mass. This will help in finding more realistic responses of different elements in the suspension system.
2. The model developed by us is for independent type of suspensions at front as well as at the rear. A model can also be developed on similar lines with independent front suspension and dependent rear suspension. This will be applicable to a large number of vehicles.
3. Imposing a few constraints related to limits on the relative velocities experienced by the subsystems of the suspension will help in analyzing the system in accordance with ISO standards.
4. Displacement of the unsprung mass at each tyre demands for the space, which is termed as suspension working space, may conflict with other features of the vehicle design. This also

puts some restrictions on the available solutions for the suspension design. Additional constraints satisfying these restrictions may find more practical solutions.

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